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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XXXI Number 6

Edited by William David Reeve

The Expediency of Compromise in Mathematical Curricula and Instruction

By A. C. Maddox
State Normal College, Natchitoches, La.

Intelligent men and women who have diligently and consistently pursued the study of mathematics and who have seriously considered the contributions of mathematics to civilization and the value of proper mathematical study to individuals find it very difficult to believe that there is real justification or educational wisdom in the current trend toward the removal of prescribed mathematics from high school and college curricula. The consciousness that mathematics is a "universal and permanent element in education" naturally causes the devotees of mathematics to deplore any movement that would give to mathematics a relatively minor place in a system of general education. The writer of this brief paper shares in a growing sense of uneasiness caused by the real or apparent lack of appreciation of mathematics that is being revealed by many educational officials in whom rests the final responsibility of determining what the various school curricula shall be. It is precisely this common uneasiness among those who place a high evaluation upon mathematical study that should lead them to make a united effort to bring about a more effective understanding between themselves as advocates of mathematical education and the school administrators as the official promoters of general education. It seems possible that such an effort would result in agreements that would produce significant benefits to individual students and to society. Let us consider briefly a few of the simpler general facts that ought to be accepted, both by school administrators and by teachers of mathematics, as basic in the serious business of advancing the highest interests of education. If such agreement as to basic facts necessitates significant concessions on the part of teachers of mathematics or of school administrators, such concessions should be forthcoming at appropriate times and places.

Teachers of mathematics should make the frank admission that much that receives wide recognition under the name of mathematical education is useless to students and detrimental to the standing of mathematics as a necessary subject of study for high school and college students. Such admission is not incompatible with the confirmed conviction that such school curricula and such teachers of mathematics should be developed as will give to mathematical study an uncontested high standing among high school and college studies. It is no doubt true that a significant part of the effort to give mathematical training to students is rendered

largely ineffective by the failure to select the most useful topics for study. It is probable that a greater part of whatever ineffectiveness inheres in the present system of mathematical education is caused by inadequate, and sometimes positively harmful, methods of teaching the various topics. It is one thing to say, for example, that the proper study of certain mathematical topics gives to students rich experiences in postulational thinking and relational thinking and sequential learning that will enhance their prospects for successful living; it is quite another thing to possess the knowledge and the vision necessary to make those same topics appear to average students anything other than convenient devices for keeping them engaged in manipulative enterprises for prescribed periods of time. Those who hold that mathematics should have a relatively large place in high school and college curricula could argue their case much more convincingly if teachers of mathematics were better qualified for their tasks.

It would be advantageous to all concerned if there were a clearer recognition of four mental attitudes toward mathematical study, any one of which a person may possess and all of which are present in typical student groups. There are some students whose instinctive or acquired interest in mathematical study is such that their pursuit of it becomes a matter of major concern to them. There are other students who develop a wholesome general attitude toward mathematics and toward a largely non-technical study of some of the philosophic aspects of mathematics or of some of the interesting results obtained through detailed investigations made by others trained to conduct such investigations. Their interest in mathematics is not primarily mathematical or vocational; it is connectional and cultural. There are some students who feel that mathematics is, or will become, a necessary tool in the pursuit of other subjects of study or of particular life vocations. They have little interest in mathematical investigations except in so far as such investigations yield formulas that will have vocational significance to them. They do recognize, however, their dependence upon mathematical techniques for much that they will need in their quest for vocational success. There are many students who apparently see no reason for mathematical study except that everybody needs a little mathematics as "a tool in everyday life." The relative number of people who regard mathematical study solely on this plane is appallingly large, which fact is probably caused largely by poor mathematical teaching but partly, also, by the fallacious educational doctrine that the only legitimate claim that can be made for mathematical study is that mathematics is a tool that is necessary for vocational living and for the mastery of other subjects of study that are vocational or cultural. The recognition of such widely different mathematical interests among students, as suggested by these four general attitudes toward mathematical study, should prompt school administrators and teachers of mathematics to give careful consideration to the respective needs of all the students in the formulation of curricula and in the teaching of mathematics; for it can hardly be urged consistently that uniform curricula and instruction should be provided for students regardless of their widely different interests and inclinations.

Teachers of special subjects are happier and more useful and probably wiser when they exhibit a reasonably magnanimous attitude with respect to the current trend toward greater freedom on the part of students in the selection of their subjects of study. Teachers of mathematics are not an exception in this matter. This does not mean that teachers should admit that it is wise for students to omit the study of mathematics from their school courses. It does imply recognition of the fact that school authorities feel bound to respect, within reasonable limits, the democratic principle as it relates to the prescription of courses for graduation from school. It also

suggests that teachers of mathematics might well accept the challenge to reconstruct their objectives and their teaching methods so that students of varying levels of mathematical interest and ability will acquire and appreciate more of the values of mathematical study. Incidentally, part of the compensation for meeting this challenge would be the privilege of dealing with students who study mathematics mainly because of genuine interest in the subject and not primarily because of official coercion. It seems fair to say in this connection that it is the duty of school officials to prescribe enough mathematical study to enable students, with the aid of wise counseling on the part of sympathetic teachers, to decide intelligently whether or not their interests and abilities justify further study of the subject. It seems fair to add, also, that suggestions, if not the advice, of well trained and experienced teachers of mathematics should be sought and considered in the construction of high school and college curricula.

If compliance with the demand for greater freedom on the part of students in their selection of subjects of study necessitates reduction in the amount of prescribed mathematics, it should be obvious to all that there is a consequent necessity for stricter enforcement of proper standards of preparation for teachers of mathematics. It is no doubt true that much of the antagonistic attitude of students toward mathematical study has been caused by poorly prepared and indifferent teachers. This unfortunate cause of difficulty will be augmented by giving students greater freedom as regards requirements for graduation unless teachers of mathematics attain to higher standards of preparation for their work, which standards pertain to the scholarship, the vision, and the temperament of teachers. The importance of the question as to what the scholastic standards should be with reference to the type and the minimum amount of mathematical training that should be required of beginning teachers of mathematics is becoming more evident as greater freedom is being extended to students in the selection of their subjects of study. It seems quite doubtful that this question should be answered without deliberate consideration of what the best qualified teachers of mathematics have to say on the points at issue. It is one thing for school authorities to rule that if mathematics is not taught in such an interesting and profitable way that students will wish to continue studying it, they must not be compelled to do so; it is quite another thing for the regulations governing the certification of teachers and the enforcement of those regulations and the practices regarding the appointment of teachers to be such as will give to mathematical education a fair chance to demonstrate what can be accomplished by a body of teachers well equipped to make mathematical study really meaningful and valuable to students.

One of the very great handicaps to the promotion of mathematical education is the widely accepted view, already referred to in this paper, that mathematics is "only a tool subject." Unless the term "tool subject" be defined so that it would be correct to regard practically all school subjects of study as tool subjects, mature students of mathematics will decline to regard mathematics solely, or even primarily, as a tool subject. Some noted authorities in matters of general educational theory and policy vigorously protest the implications of this restricted view of the values of mathematical study. Dr. Charles H. Judd says, "The man who calls arithmetic a 'tool subject' and with this name dismisses it as something less worthy than subject matter courses is guilty of criminal neglect to true values." He says further, "I am told that the school should teach children how to measure wall paper and how to tell time and that sections of arithmetic should be devoted to these specific tasks, but I look in vain for any appreciation of the fact that the school ought to lead pupils who have only a hazy and unsys-

tematic notion of the world to see the value of arrangement and order in all thinking and to cultivate the general ideas of regularity and precision." It is unfortunately true that many teachers of arithmetic and of other mathematical subjects think and talk about these subjects merely as "tool subjects" and many other teachers use only such teaching materials and methods as would indicate that their conception of the usefulness of mathematical study ends with the "tool subject" idea. While every devotee of mathematics knows that mathematics is one of the indispensable tools in the development of numerous other branches of knowledge and in the development of civilization and in many of the ordinary situations of daily life, there should be a renewal of individual and concerted effort to lead increasing numbers of people to recognize and appreciate mathematics, and particularly the number system, as "a mode of thinking" that "pervades every observation which modern man makes." Students who develop this broader view not only derive greater enjoyment from their mathematical study, but they acquire, even though somewhat incidentally, a more thorough and more effective mastery of the "tool use" of mathematics than can possibly be acquired by students whose only real interest in mathematical study is prompted by their need of certain mathematical facts to be used merely in the achievement of some purely vocational goal or in meeting some simple "everyday life situation."

If devotees of mathematics are in earnest in their claims as to the potential values of mathematical study they should seek to bring about among themselves and among school administrators a concerted effort to make the revisions that are necessary in the content of mathematical courses, in methods of teaching, and in techniques for testing, in order that the total program of mathematical education may be brought into maximum harmony with the major claims made by these devotees. For example, if the claim is made, as it may well be made, that the proper mode of mathematical study makes larger demands on the reasoning ability of students than upon their memories, the materials of instruction, the class-room procedures, and the testing devices should be such as to emphasize the correctness of the claim. Or if it is claimed that the development of mathematical power is a far more worthy objective than the mere acquisition of mathematical knowledge, as the best mathematical educators actually do claim, consistent effort should be made by those who determine material content, teaching practices, and testing programs to lead students, by means of all of these, to develop the ability and the inclination to conduct independent investigations in their mathematical study. Furthermore, there should be continuous and systematic effort to educate the public, and particularly the school people, to understand and to appreciate the fact that the value of studying a particular body of subject matter is not always proportional to the amount of that subject matter which is "remembered" by the student. In such connections as these, it cannot be too strongly emphasized that only those who are in substantial accord with the major objectives of those who do the actual teaching can formulate proper materials and devices for determining whether or not those objectives have been achieved, as it must be obvious that there should be harmony between the main goals to be achieved by any enterprise and the means of testing the effective results of that enterprise. This emphasizes the importance of a better understanding among school administrators, supervisors, and teachers of mathematics as to the principal reasons why mathematical education should be given a fair chance to play a relatively important role in the promotion of general education.

If there is much truth in what has been said in this paper, there should be brought about a better understanding and a more

effective cooperation between representatives of the great body of teachers of mathematics and the officials who are charged with the final responsibility of formulating curricula and of making general provision for their effective use in the schools. While it is not, and will never become, necessary for any person or group of persons to "defend mathematics," it is the common desire of those who know the "practical" or educational values of proper mathematical study that all possible effective steps shall be taken to the end that such values may be more widely realized and appreciated. If this worthy desire is to be fulfilled in any large measure, there must be a renewal of faith in the power of mathematics as "an energizing principle." There must be also a bold acceptance of the challenge that comes in this time of popular demand that those who advocate the inclusion of any subject of study in the program for the general education of the people must give convincing evidence of the validity of the claims which they make for that subject. If all who are in any way responsible in this important task as it relates to mathematical education will fulfill their obligations with conviction and energy, the results will be genuinely gratifying and the true end of education will be more nearly achieved.

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The New Arithmetic Course in the Chicago Elementary Schools—A Criticism

By HARRY G. WHEAT
Professor of Education, West Virginia University

IN a recent issue of THE MATHEMATICS TEACHER the new arithmetic course in the Chicago elementary schools is characterized by its sponsor[1] as having as its theme "Teaching Arithmetic with Meanings." In support of the characterization three departures from earlier practices are described:

> (1) All formal arithmetic requirements have been taken from grades one and two. There are no definite drills for memorization and mastery of the addition and subtraction combinations, in order to attain automatic recall, definite goals, or definite time limits. . . .
> (2) The difficult aspect of each arithmetic topic has been moved up to a grade placement as far as distribution in grades allowed. . . .
> (3) . . . Where now there is sufficient experimental evidence for the superiority of one method over another, the superior method will be recommended. . . .[2]

The reader of the article who is in search of meanings for arithmetic is left to wonder where are the meanings and what is the source of the meanings that are claimed for the new Chicago course. From the article he has to turn away empty handed. He is forced to conclude that the new arithmetic course in the Chicago schools either is inadequately described by its sponsor or else is very narrowly conceived. Moreover, he is made somewhat impatient with the thought that many a casual reader, who does not look beyond the claims that are made, will conclude that such a course in arithmetic as is described sets a pattern for the provision of meanings.

It is from this point of view that the three departures claimed for the new Arithmetic Course in Chicago may be analyzed.

1. POSTPONEMENT OF ARITHMETIC STUDY

In the first characterization of the new course, indicated above, there is no suggestion of the provision of meanings. Instead, one is presented with a statement that seems contrary to fact, followed by one that is both repudiated by experience and explanatory of much of the confusion that exists in primary number work. The first statement to which reference is made is given in support of the postponement of formal requirements. It is to the effect that primary children experience great difficulty in learning the sequential order of the number names and the distinction between cardinal and ordinal number. This is offered apparently in all seriousness, but in direct contradiction of the author's earlier statement of the child's ability in number when he enters school for which there is an abundance of supporting evidence:

> Long before he has entered school the child has had many, though limited, experiences with numbers in his everyday normal activities. *He has counted* members of his family, his toys, his age. . . .[3]

Anyone who can count, though he can go no further than two, or three, or four, has no difficulty with the distinction between cardinal and ordinal numbers. The essence of counting is its system and its fixing of attention upon the group being counted as a whole. As he counts, one, two, three, each number name is used in the double sense. "One" means the first one as well as all counted up to that point;

[1] William H. Johnson, "The New Arithmetic Course in the Chicago Elementary Schools." THE MATHEMATICS TEACHER, XXXI, (March 1938), pp. 126–128.

[2] P. 127.

[3] P. 126. The italics, here and elsewhere in the quoted statements, are the present writer's.

"two" means the second one as well as the second combined with the first. "Three" means both the third and last one counted and the whole group thought together. Each number name, though it is the cardinal name, has a double meaning. The child who counts deals with both ordinal and cardinal numbers at one and the same time. Practice in counting helps to make distinction as well as relation clear. It is certainly stretching the point to argue that the child's difficulty with cardinal and ordinal number justifies a two-year postponement of arithmetic. How can one note such difficulty on the part of the child of five who relates, "There were three of us. I wanted to play with the wagon first, but I had to be third"?

The second statement to which reference is made is as follows: "When the above concepts and procedures [namely, the distinctions between cardinal and ordinal number] have been woven into experiences that come naturally into the life of the average child, *he is ready for formal addition and subtraction.*"[4] This is the statement that experience repudiates, and it is reliance upon this statement that explains initial failures in number work.

The failures in arithmetic, to which the author has pointed as disconcerting facts, usually begin with the introduction of the additions and subtractions. No great difficulties are experienced in learning to count. Those pupils who have not already made a beginning in counting before they come to school, and those whose earlier beginnings have been somewhat confusing, respond quickly to the instruction in counting that most teachers seem prepared to give. Their success at this first step in number seems to forecast a fair measure of success at succeeding steps. Generally, however, the next step that is indicated to the pupils is the step where failure begins. This is not due to inherent difficulties in addition or to the native inabilities of the pupils. It is due to the facts that addition is more than one step removed from counting and that counting does not adequately prepare most pupils for addition. Counting is a means of apprehending small groups. It is not the only means. Other means that grow out of and relate to counting, that are intermediate steps between counting and addition, have to be indicated to many children and taken by them before they are ready to take the step of addition. To add understandingly, that is, "with meaning," one must have a clearer idea of the groups to be counted together than counting alone is able to provide. A two-year postponement of addition is not merely unnecessary; it of itself is no provision for the intermediate steps between counting and addition. Those pupils in the Chicago schools who are found to be ready for addition when it is introduced in the third grade will be found to be those who have taken the intermediate steps as incidents of their experiences outside the arithmetic course provided for them and largely by accident, but not as a result of any apparent provision in the course, as described, for a systematic portrayal of the steps.

[4] P. 127.

2. Grade Placement of Arithmetic Topics

In the second characterization of the new course provision for meanings is again not suggested. Instead, official sanction seems to be given to the easy way of solving difficulties by shoving them along to the teacher in the next grade. There is nothing new in this procedure. It has been used by teachers, without official authorization, to be sure, long before the Chicago course and the "experimental evidence," to which reference is made in the article, gave it approval. All of us have observed division, for example, as a topic of instruction in the sixth, seventh, and eighth grades, and the procedure of instruction of pupils in these grades, who had not yet learned division, moving along with quite as much unsuccess as ever characterized instruction in the same topic of younger

pupils in earlier grades. We have observed, too, that it is not the pupil's age, height, weight, or location in the school, that determines his success in learning division, but his acquaintance with and understanding of the mental processes which precede and lead up to division and which, with the teacher's stimulation and guidance, can carry him along through the procedures of division. As indicated in the foregoing paragraph, the mere postponement of a topic has nothing at all to do with the provision of a meaningful setting for it. If one postpones a topic and then continues to teach it as he always has, he should expect no more successful results than he formerly secured.

3. Choice of Methods

The third characterization of the new course leaves the provision of meanings a matter of considerable doubt. One should seriously question whether meanings in arithmetic are going to be provided by an improvement in methods of instruction; whether what we teach in arithmetic is not a prior consideration to the methods we decide to use; whether there is any one best method of teaching anything in arithmetic. If, as suggested, "formal arithmetic" is to be begun in the third grade, and "formal arithmetic" is "definite drills for memorization and mastery,"[5] one may well inquire "what difference does it make how much methods of teaching are improved?" The drill method of instruction has never yet provided meanings. Can movement be made in the direction of meanings by teaching unsuccessful procedures in more successful ways? Again, can meanings be provided the better if what the pupil is to be set at learning is to be "reduced," as suggested, in terms of what "pupils really need?" Pupils really need little, if any, arithmetic until they acquire it as useful and understandable ways of thinking, just as industry had no need of the induction coil until Faraday made it available. Moreover, if meaning is thought to inhere in a kind of arithmetic that can be called meaningful, should we not expect more meanings to be secured by teaching more, and not less, arithmetic? Finally, if it is possible to decide upon meaningful methods of thinking in arithmetic and to plan instruction in them rather than in meaningless rule-of-thumb procedures, would it not be advantageous to present and describe and illustrate such meaningful methods in as many ways as possible? Would not such multiple presentation of the many-sidedness of a meaningful method of thinking be more profitable than any single "superior method" of instruction?

The writer of the foregoing criticisms is not familiar with the operating details of the new arithmetic course in the Chicago Elementary Schools. He, therefore, would hesitate to suggest that the criticisms are applicable to the course that is actually in operation. He does submit, however, that they are applicable to the *description* of the course as an "arithmetic with meanings."

There may be readers of The Mathematics Teacher who have not seen anything in the description of the Chicago course that even faintly suggests an "arithmetic with meanings." To such, the criticisms may very well appear as useless attacks upon a straw man. There may be other readers, however, who have been inclined to take the description at its face value. If the distinguished superintendent of the Chicago Schools so offers his description, may it not be expected that many in humbler positions will be inclined to view it eye to eye with him? To such readers the foregoing criticisms are addressed.

[5] P. 127.

The National Council Committee on Arithmetic

By R. L. Morton
Ohio University, Athens, Ohio

In recognition of a growing demand for a committee, nation-wide in its representation, to study and report upon trends in the arithmetic curriculum and in the teaching of arithmetic, The National Council of Teachers of Mathematics in February, 1937, authorized the appointment of a committee on Arithmetic.

The Committee is composed of 12 persons. They are: H. E. Benz, Ohio University, Athens, Ohio, *Secretary;* E. A. Bond, State Teachers College, Bellingham, Washington; William A. Brownell, Duke University, Durham, North Carolina; Leo J. Brueckner, University of Minnesota, Minneapolis, Minnesota; G. T. Buswell, University of Chicago, Chicago, Illinois; Paul R. Hanna, Stanford University, Stanford University, California; Julia Hodgson, Peabody College for Teachers, Nashville, Tennessee; R. L. Morton, Ohio University, Athens, Ohio, *Chairman;* Lorena B. Stretch, Baylor University, Waco, Texas; B. A. Sueltz, State Normal and Training School, Cortland, New York; C. L. Thiele, Public Schools, Detroit, Michigan; and H. G. Wheat, West Virginia University, Morgantown, West Virginia.

The Committee held its first meeting in conjunction with the convention of the National Education Association at Detroit in June, 1937. At that time, the appointment of a sub-committee of three was authorized to compile a statement of basic points of view as to arithmetic and arithmetic teaching to which the Committee could subscribe. This sub-committee was instructed also to assemble a list of questions pertaining to important issues in arthmetic course of study organization and arithmetic teaching. Each of the members of the Committee was requested to submit his suggestions as to basic points of view and questions to the sub-committee. The sub-committee consisted of Mr. Buswell, Mr. Thiele, and Mr. Morton.

At a second meeting of the Committee, held at Atlantic City in February, 1938, the report of the sub-committee was considered and revised. The report is presented herewith. It is the hope of the Committee that this statement will be read critically by the educational public in this country and that reactions to the statement will be expressed. It is not intended that this statement should include all of the important issues as to arithmetic and arithmetic teaching but that it should afford the Committee an opportunity to express opinions on some of these issues and to propose questions.

The nature and function of arithmetic. There are many situations of life in which order, magnitude and quantity are essential elements. Arithmetic is a systematic pattern of thinking about such situations. It is also a series of modes of attack upon them. It is thus more than a tool which one may use in the interpretation of personal, business, social, or civic affairs. It grows out of a productive study of such affairs and is applicable to them. At every point in its development it is intimately related to them.

The functions of instruction in arithmetic, then, are to teach the nature and uses of the number system in the affairs of daily life and to help the learner to utilize quantitative procedures effectively in the achievement of his purposes and those of the social order of which he is a part. This conception recognizes two major mutually related and interdependent phases, namely, the *mathematical* and the *social.* Both phases are essential. Neither by itself embodies appropriate instruction and neither will develop in the minds of the pupils the arithmetic that they should learn. Mere grasp of the

mathematical phase does not provide for completely social significance; and, conversely, a too exclusive emphasis upon the social phase will not guarantee grasp of the mathematical.

The purposes of stressing the mathematical phase are: (1) to develop the ability to recognize among all the attractive and objective elements of appropriate situations the number element that does not meet the eye but, nevertheless, gives them their order and exactness; (2) to introduce pupils to the systematic methods of attack upon the number elements of situations that have finally been brought together into the unified number system as we know and use it today; (3) to develop in pupils both confidence in the reliance one can place upon such methods of attack and facility in their use by guiding practice in number thinking to higher and higher levels of effectiveness; and (4) to train pupils to carry to later and more advanced studies of important personal, business, social, and civic situations these methods of attack with the deliberate purpose of analyzing out their number elements and determining their relations to the other elements which with the number elements comprise the whole.

The purposes of stressing the social phase are: (1) to insure understanding of the contributions number has made to the development of the social institutions that have made possible the progress of the human race; (2) to insure that the work of pupils in arithmetical processes will have meaning and significance; and (3) to develop in pupils the disposition and ability to apply increasingly mature procedures in the quantitative situations of life.

Views of arithmetic. The purposes of arithmetic, as these have been outlined above, imply certain theories regarding content, instruction, and measurement—theories which are by no means in accord with various current views and practices.

The drill theory. According to one current view, arithmetic, for the purposes of learning and teaching, may be analyzed into a great many units or elements of knowledge and skill which are comparatively separate and unconnected. The pupil is to master these almost innumerable elements whether he understands them or not and is to learn them in the form in which he will subsequently use them. The pupil will attain these ends most economically and most completely through formal repetition.

This view, which for convenience may be called the *drill theory*, is not so popular in 1938 as it was a decade or two ago. Certainly no competent student of arithmetic would knowingly advocate its general adoption. Nevertheless, many practices still common in classrooms testify to the fact that the drill theory has not been entirely abandoned.

To the drill theory of arithmetic this committee takes vigorous exception. In the first place, the drill theory neglects the social aim of arithmetic. Arithmetic taught in a vacuum can hardly be expected to prove useful as a means to happier and more intelligent social participation. In the second place, the drill theory must also inevitably fail to serve the mathematical aim of arithmetic, no matter how assiduously children are "drilled" in the supposed elements of the subject. Repetition and unintelligent abstract practice cannot yield the kind of quantitative understanding and insight which this committee believes to be essential to true arithmetical learning.

The incidental theory. A second view of arithmetic from which the committee dissents may be called the *incidental theory*. According to this theory children will learn as much arithmetic as they need, and will learn it better, if they are not systematically taught arithmetic. The assumption is that children will themselves, through "natural" behavior in situations which are only in part arithmetical, develop adequate number concepts, achieve respectable skill in the fundamental operations, discover vital uses for the arithmetic they learn, and attain real pro-

ficiency in adjusting to quantitative situations.

The incidental theory seems to be a natural reaction against the older drill theory, in that it emphasizes, as the latter does not, the social significance of number. On the other hand, this theory, while it should contribute to the social aim of arithmetic, is necessarily one-sided: it disregards the equally vital mathematical aim. Taught incidentally, arithmetic cannot develop in children systematic methods of dealing with the ever increasing number of quantitative problems in our society; at best it can but fit children to deal more or less effectively with the few types of quantitative problems which arise during the school day.

The committee's position. As stated in the first section of this report, the committee stands for a kind of arithmetic in which both the mathematical and the social aims are clearly recognized—and clearly recognized as interdependent and mutually related. Attainment of the mathematical aim is regarded as possible only if meaning, the fact that children shall see sense in what they learn, is made the central issue in arithmetic instruction. Arithmetic is conceived as a closely knit system of understandable ideas, principles, and processes, and an important test of arithmetical learning is an intelligent grasp upon number relations together with the ability to deal with arithmetical situations with proper comprehension of their mathematical significance.

In the teaching of arithmetic there is place for practice, but not for mere "drill" (which, almost by definition, consists in the repetition of ideas and skills at whatever level of understanding has been attained). Practice of appropriate nature encourages growth in thinking at steadily rising levels of abstractness; it fixes for retention what has been learned, and it increases the facility with which it can be used.

Likewise, in the teaching of arithmetic there is place for all the incidental number experiences which occur in unpredictable ways in the schoolroom and outside. The wise teacher will employ these incidental occurrences of number to their fullest possible extent, the better to reveal the usefulness of arithmetic. But the wise teacher will not confine herself to the incidental occurrences of number in social settings. Rather, recognizing the importance of the social aim of arithmetic, she will actively seek and plan other social situations of a quantitative character. She will lead her pupils to see the number imbedded in these situations; she will use the situations to motivate the learning of new skills and ideas; and she will help her pupils to develop the disposition to use the arithmetic which they have learned. In a word, she will show arithmetic to be both socially significant and mathematically meaningful.

Statements and Questions

The assumption of any one of the three positions which have been stated leads naturally to more specific statements. While there is always some hazard in reducing general principles to specifics, the Committee is submitting the following list of statements in the belief that such a list will be more likely to evoke constructive criticism than will the bare formulation of principles. The statements which follow are classified under four general headings, namely: (1) selection of content; (2) organization and grade placement; (3) methods of teaching; and (4) measurement and evaluation. Following each section is appended a short list of questions.

1. *Selection of content.* (a) Arithmetic is an important means of interpreting children's and adults' quantitative experiences and of solving their quantitative problems. Consequently, the content should be determined largely on the basis of its social usefulness and should consist of those concepts and number relationships which may be effectively used.

(b) The Committee believes that the

social-survey techniques commonly employed to determine the content of arithmetic are not adequate. This procedure merely selects the arithmetic that *is* used, but gives no indication of what arithmetic *might be* used to advantage. Likewise, the analysis of the processes of arithmetic into component elements for teaching purposes fails to provide the meanings and relationships which should be the core of learning.

(c) Mathematical processes have a high degree of permanence and coherence, but in their social applications certain topics may deserve more emphasis in one generation than in another. The basic mathematical concepts and processes remain the same from one generation to another, but the application and utility of them may vary.

(d) One effective means of showing children the social significance of arithmetic is to acquaint them with the history of number and with the contributions of number to our civilization. This aspect of arithmetic has been much neglected.

(e) Intelligent mastery of the fundamental processes of arithmetic is dependent upon a thorough grasp of (1) the meaning of number as such, (2) the meaning of our number system, (3) the meaning of the processes by which numbers are related, and (4) to some extent, the meaning of the forms used in computation.

(f) Under the influence of the drill theory, the computational skills of arithmetic have been overemphasized. More attention needs to be given to the non-computational values of arithmetic, such, for example, as are found in the quantitative thinking which should accompany intelligent reading and listening.

Questions relating to content. (a) Have the logical nature and sequential development of the facts and processes of arithmetic been too prominently featured in courses of study? Have such other factors as pupil maturation and social importance of materials been sacrificed?

(b) Is there an essential distinction between "concrete arithmetic" and "abstract arithmetic"?

(c) Is our present adult population too dependent upon paper and pencil for the simpler calculations? If so, is this because the teaching of arithmetic has neglected judgment, evaluation, and estimation?

(d) Repeatedly, teachers report that children in the lower grades take delight in learning abstract facts such as number combinations. Would it be wise and sensible to capitalize this type of learning?

(e) Such ideas as "proportion" and "average" have value as forms of thought. Should more emphasis be given to these and similar phases of arithmetic?

(f) What differences in arithmetic content may well be made for the slow and the bright?

(g) What values stressed by writers on arithmetic who entertain different conceptions of the subject are apt to be lost if the Committee's position is generally adopted?

(h) To what extent should geographical and environmental variations affect the learning program?

(i) What mathematical values other than those resulting from a study of numbers (arithmetic) should be given a place in the elementary school?

2. *Organization and grade placement.* (a) Surveys of the number experiences of young children indicate that they have had considerable experience with number before the age of entering school and that they are ready for arithmetic. At this stage, however, abstract symbols and processes should be used as records of number experiences and, in general, should be postponed until after basic understandings have been secured.

(b) In general, a rich conceptual background should be developed before the manipulative phases of a topic are learned. A pupil should appreciate and understand the meaning and significance of what he is attempting to learn. Usually, his environmental experience is a potent factor in his appreciation and understanding.

(c) To learn the social values of arithmetic, children should engage in appropriate activities. These activities, however, must not be haphazard experiences. The experiences should be organized so as to provide an orderly growth in the understanding of number and its use.

(d) Growth takes place slowly and enriched ideas result from a multiplicity of varied experiences. Such growth requires an expansion or stretching out of topics rather than a placement of topics as wholes according to grade or mental age.

Questions on organization and grade placement. (a) What is a desirable time allotment for arithmetic?

(b) Can new learning be based upon social experiences of children? To what extent should new learning be based upon the logical relationships of arithmetic?

(c) Is there a place for a systematic course in arithmetic in the senior high school? If so, what should be its nature and content?

(d) By what criteria can the grade placement of topics be determined? For example, should the systematic study of measures and weights, and of the relations among them, be delayed until pupils have developed an understanding of the number ideas that give precision to these relations, or should this systematic study be used to give significance to number ideas?

(e) Is the current tendency to stretch topics over several years educationally sound?

(f) What can be done to adjust the conflict between the sequential nature of arithmetic and promotion on the basis of age?

3. *Methods of teaching.* (a) To be most valuable socially, arithmetic must be taught with the aim of developing resourcefulness in dealing with quantitative situations. This will not be achieved from mechanistic learning nor from emphasis upon the social values of arithmetic. Resourcefulness may be achieved by pupils who constantly utilize meaning and understanding in their dealing with numbers.

(b) Practice on basic processes contributes but little to the enrichment of meaning since it represents a series of recurring experiences of the same or similar kinds. However, practice is useful if introduced at the proper time and in proper amounts. Practice, or drill, should be used only after the thing to be drilled upon has been thoroughly understood.

(c) The separate arithmetic period is justified by the opportunity it affords to teach the meaning of arithmetic, but the limitations of the special period, from the standpoint of teaching the significance of number, must be fully recognized. This same type of limitation is encountered in a departmental organization. In general, the integration of arithmetic with other school activities seems to be best accomplished in a non-departmentalized organization or in one in which very specific provisions for integration are set up.

(d) The proper use of activities provides an excellent opportunity for effective teaching. The Committee is not critical of the use of activities in a systematically organized program. It is critical of unplanned, haphazard activities which vary from day to day according to the suggestions of pupils rather than according to the needs of a coherent and clearly-thought-out program of arithmetic.

(e) Regardless of which methods of teaching and learning are employed in the encountering, recognizing, and learning of arithmetical materials, the teacher should be conscious of and able to recognize desirable arithmetical concepts, principles, and processes in social and economic situations.

Questions on methods of teaching. (a) What should be the function of the teacher—to develop methods of discovery and learning which the pupils may use to determine results for themselves, or to demonstrate the procedures which the pupils then learn and use?

(b) In light of new points of view rela-

tive to arithmetic, what is the function of practice?

(c) Can the important concepts, principles, and skills of arithmetic be acquired economically and readily in an activity program?

(d) When should the symbols and forms (algorisms) of arithmetic be introduced—before, after, or while the meaning and use of each are being learned?

(e) To what extent can pupils in the beginning grades learn methods of work and employ them in determining results independently? That is, must pupils at the outset learn the "number facts" on the teacher's authority, or can they be taught to arrive at them on the authority of methods which they can learn to employ?

(f) What should be the real function of the usual type of verbal "problems"—to introduce pupils to methods of thinking which are useful in the discovery of truth, or to provide practice in the use of ideas that have been gained by means other than problem-solving?

(g) What variations in learning should be encouraged for pupils of varying abilities?

4. *Measurement and evaluation.* (a) Measuring the results of arithmetic taught according to the point of view expressed in the preceding paragraphs can not be accomplished by the use of tests designed for measuring the results of drill. At the present time, there is a conspicuous lack of tests suitable for measuring arithmetical and quantitative thinking in non-computational situations.

(b) Speed norms should be interpreted in terms of the modes of learning and the normal reaction rates of individual pupils. In general, speed has been overemphasized at the expense of adequate understanding.

(c) Diagnostic tests are important instruments in teaching. Growth in quantitative thinking is best observed in the mental processes which children employ rather than in the correctness of the answers which they secure. While correct methods must eventually result in correct answers, it is desirable that in the process of learning the teacher be provided with tests which will enable her to identify the various degrees of understanding as the child's learning matures. This means that group instruction and group testing must be supplemented by individual instruction and individual testing.

(d) Individuals may be expected to differ materially in the ease with which they can acquire expertness in quantitative thinking. The measurement of these differences should result in the construction of varied materials and the development of varied methods of instruction.

Questions on measurement and evaluation. (a) Have the schools during the past century unduly emphasized speed in computation? By what criterion should the reasonableness of speed in a given process by pupils in a given grade be guaged?

(b) To what extent would pupils be helped if, instead of emphasizing speed, the teacher would teach them how to be thoughtful, how to take their time to think a process through, how to be deliberate? Is not deliberation more of a human process than machine-like speed?

(c) What should be the criteria for selecting and weighting test items?

(d) What hitherto neglected phases of arithmetic ought to be considered in the development of new testing programs?

(e) What are the essential features of an effective program of diagnosis and remedial teaching?

Conclusion. The Committee invites comments and criticisms as to its position. Naturally, the statements which the Committee has offered are tentative and subject to modification in the light of the criticisms which the Committee hopes to receive. In inviting criticisms of its position, the Committee is desirous of attempting to reach some theoretical statement upon which the teaching profession may agree and which may serve as a basis for a unified effort to improve the program of arithmetic.

The Teacher's Professional Study of Subject-Matter in Mathematics*

By WILLIAM C. BAGLEY
Teachers College, Columbia University

I

I AM somewhat handicapped in dealing with the subject assigned to me on the program since I discussed certain phases of it in a paper that I presented before this Council in 1933.[1] While few if any of you will recall that paper, it is a matter of record in your journal and I dislike under these conditions to repeat what I have already said. I may be permitted, then, to outline very briefly here some of the considerations that led to the development of the conception of the professional treatment of subject-matter as a central feature in the education of teachers, and to set forth some of the implications of this conception as applied to the preparation of those who are to teach mathematics as a phase of general rather than of specialized education.

The qualification just suggested is of prime importance. Let us consider a teacher who is to teach his subject only to selected students who are highly capable and who have a definite interest in the subject and perhaps also a vocational goal in preparation for which a mastery of the subject is essential. Speaking generally, the professional preparation of such a teacher beyond a thorough training in the subject itself is relatively unimportant. He has himself traversed the road and he has but to guide the learner over the same route. By hypothesis, the learner is "selected." This means that he compares favorably with the teacher in native mental ability and that he has a powerful motive for putting forth the effort that mastery involves. If the student does not meet these conditions, he should in traditional theory look elsewhere, and in the practical administration of the high schools and colleges he has until recently been forced to look elsewhere. No need was felt to adapt materials or methods of teaching to the capacities of the average mind.

It is not difficult, then, to understand why the broader professional education of teachers has been a relatively late development. For only a little more than a hundred years has formal education, even on the most elementary levels, been provided for large unselected population-groups. The universal primary school even in the most advanced countries was practically unknown before the nineteenth century, and until the universal school came into being, the preparation of teachers to adapt instruction to heterogeneous groups was neither sufficiently important nor sufficiently necessary to create a demand. The development of strictly professional schools for teachers paralleled the development of the universal primary school, and it is significant of the relatively recent development of the universal school in our courty that a year still remains before we shall celebrate the one-hundredth anniversary of the first State normal school, established at Lexington, Massachusetts, in 1839.

The upward expansion of the universal school into the secondary field did not have its first feeble beginnings for another fifty years. Not until the last decade of the 19th century did the high school even promise to become non-selective and not until then did the professional education of high-school teachers beyond the mastery of their subject-matter become a recognized need. After the turn of the

[1] "The Ideal Preparation of a Teacher of Secondary Mathematics, from the Standpoint of the Educationist," THE MATHEMATICS TEACHER, May, 1933, pp. 271 ff.

* A paper prepared for the 19th Annual Meeting of the National Council of Teachers of Mathematics, Atlantic City, February 25, 1938.

century this movement was rapidly accelerated. The vast expansion of the American high school—a growth unprecedented in history and unparalleled in other countries—was clearly in evidence by 1910. Its acceleration continued and the rate of acceleration increased until today secondary education approaches universality more nearly than did elementary education two generations ago.

From its beginning the non-selective secondary school was faced with the problem of adapting instruction to widely varying needs and capacities—to adapt its materials and its methods to the mass-mind. To prepare teachers who could solve this problem in some measure was the demand which the now non-selective high schools laid upon the colleges and universities from which they were recruiting most of their teachers.

Hesitatingly and almost always unwillingly the colleges attempted to meet this demand. They did it in their traditional way by appointing professors of education who would offer courses designed to make high-school teachers better teachers. There were about a score of these professors of education in 1900, but with increasing pressure from the high schools and with the development of graduate work in education in Clark University and the University of Chicago and Columbia and New York universities just before the close of the century; some of the State universities soon after; and a large number by 1920, professors of education multiplied rapidly and the university study of school administration and the problems of teaching soon had an established—although, in the eyes of the academicians scarcely a respectable—status.

In the early efforts to solve the problems of the teacher in the non-selective high school, the usual plan was to provide in the junior year a course in what was sometimes called "general method," sometimes "principles of teaching." This plan assumed that definite laws or principles determined the effectiveness of teaching and that if prospective teachers were properly instructed in these laws or principles they would apply them readily enough to the teaching of any subject in the lower schools. These general-method courses commonly had as a prerequisite a course or courses in psychology, then vigorously developing as an empirical and experimental science upon the analogy of physics and physiology. The line of reasoning was natural enough. Just as modern engineering had been built upon the tested principles of mathematical and experimental physics and just as modern medicine had been built upon the tested principles of chemistry and physiology, so the practice of teaching could be built upon equally valid principles derived from scientific psychology. It was truly an alluring prospect, and many of us who came into the field of professional education at that time were convinced that it would be only a few years before our problems could be reduced to law and order and predictive certainty.

It was not at all so simple, as subsequent events have proved. The professional education of teachers has some analogies with the professional education of engineers and physicians, but almost always the more obvious an analogy, the more treacherous it may be as a guide to action. It required many years clearly to demonstrate that the application of objective and experimental methods of investigation to mental and social phenomena encounters far greater difficulties than the application of these methods in the fields of physics and biology. Today one must reluctantly acknowledge that the term "general method" as applied to teaching procedures is a misnomer, and that the term "principles of teaching" does not connote anything approaching the same certainty, accuracy, and reliability in prediction and application as do such terms, say, as Boyle's Law or Mendel's Laws.

But all this minimizes in no measure the imperative necessity that the teacher adapt both his methods and his materials of instruction to the needs and capacities of unselected groups of pupils and stu-

dents. Nor does it mean that an analysis on his part of this basic problem and an acquaintance with ways in which others have worked toward its solution may not be a helpful part of his professional education. Our early hopes based upon an oversimplification of the problem have been disappointed, but under existing conditions this fact should be a distinct and stimulating challenge to renewed effort.

One of the most promising leads that may profitably direct such effort is the conception of the professional study of subject-matter. This conception rests upon the hypothesis that both effective methods of teaching and valid criteria of selection will vary widely in the different subject-matter fields. Consequently a truly professional preparation for the teacher cannot be achieved merely by requiring him to take detached and general courses in education in the hope that he will later attach them helpfully to the subject-matter that he plans to teach. It rather involves an intensive study of his subject-matter in intimate connection with the problem of teaching appropriate parts of it to others. The specific selection of those parts and the specific methods of teaching them to heterogeneous groups of learners are primarily bound up in the subject-matter itself. They must be derived from the subject-matter rather than imposed upon it from without.

It seems obvious, furthermore, that the whole process must be directed by one who is a competent master of the subject field rather than by a professor of education. But this does not mean that the subject-matter professor who is to essay this task of direction can succeed if he is content to be merely a competent scholar in his field. In addition to this scholarly equipment he must understand and appreciate the difficulties and the significance of mass-education as opposed to selective education. The professor of education alone and unaided has not been able to solve the problem of mass-education but he has recognized that it is a basic problem at the present stage of social evolution. Nor have his efforts toward its solution and especially his investigations of its difficulties been entirely futile. One who would teach his subject professionally must have a similar understanding and appreciation, and he must be as well acquainted as possible with what the educationist has accomplished. The advocate of the professional treatment of subject-matter maintains that it is a simpler matter for certain specialists in the various subject-matter fields to acquire a working knowledge of educational theory and a working mastery of what educational investigation has discovered than it would be for the educationist to acquire a knowledge and a mastery of all the specialized fields of scholarship with which mass-education must be concerned.

II

Since I speak with the limitations of a layman in mathematics, I can only refer in general terms to those features of instruction in mathematics designed for prospective teachers that seem particularly pertinent from the educationist's point of view.

In the first place, I take it for granted that every teacher should have a far wider knowledge of his subject-matter field than he will normally be called upon to teach. What branches or divisions of mathematics this background of mathematics should include is obviously to be determined by scholars in mathematics and preferably, I venture to say, by scholars in mathematics who know something of the needs and problems of general education. In other words, in the program of instruction for mathematics teachers, these background courses are themselves professional since they are essential parts of the teacher's professional equipment. That they may in certain cases be identical with courses that meet in addition other needs —such as "general culture" or preparation for professional callings like engineering— does not alter the fact that in the sense in which we are using the term they are professional for teachers. In order to keep the

professional objective to the forefront, I refer to them as "professional-background" courses.

I am particularly concerned, however, with a more specific type of professional instruction which, taking advantage of this broad background, will deal in detail with the problems of adapting subject-matter and developing methods of teaching for the courses in mathematics which the student will later teach either in the lower schools or in some other field of general, non-selective education such as the junior college or the adult classes which are now multiplying so rapidly. These highly specific courses I call "professional-content" courses to distinguish them from the broader "professional-background" courses just referred to.

Here a most important problem would apparently be first to determine what mathematical materials may most profitably be made part of the pabulum of general education. The development of courses in general mathematics on the junior-high-school level has, of course, been attempted for some time, with how much success I am not at all qualified to speak, but the effort is at least a recognition of the need that I have in mind. One may safely assume that there are certain basic concepts of mathematics which, with appropriate instruction, can be grasped by the average mind, and which will be of sufficient value to the average mind to warrant the necessary expenditure of time and effort. Mathematics has played so fundamental a role in human progress that to understand and appreciate at least some of its contributions is essential to an understanding and appreciation of the civilization which mathematics has done so much to make possible. To insure this understanding and this appreciation in all major subject-matter fields is in my judgment one of the basic functions of general education as distinguished from specific or vocational education.

If this position is valid an important phase of the teacher's professional preparation in whatever field he may teach, and especially in mathematics, should involve an acquaintance with the history of his subject, including not only a chronicle of its development—an account of how knowledge has been made in his field—but also an appreciation of the influence that this knowledge has had upon human progress. Here the role of mathematics has been so central and so profound that its story properly told reaches not merely dramatic but truly epic proportions. Here, too, the human interest can be profitably and legitimately capitalized both in the story of pure mathematics, as Mr. Bell[2] has recently so well demonstrated, and in the relation between mathematics and such relatively recent events as the spectacular development of radio.

Lest it be inferred from what I have just said that I should restrict mathematical instruction for the masses merely to a development of those concepts and principles that can be easily understood and in which an interest may be easily aroused, I hasten to add that in the high schools and the junior colleges and the adult classes every effort should be made to introduce as large a proportion of the learners as possible to the systematic and sequential study of mathematics. I respect the charitable motives of many educationists who would keep pupils and students from attempting work in which they may experience failure. These critics are particularly insistent that the exact and exacting studies such as mathematics and physics and Latin are suited only to those whose I.Q.'s are in the higher brackets and even do not "function" (as the favorite expression has it) in the lives of many of these. It is a plausible argument and can be supported by the appealing testimony from the field of educational psychology anent the alleged fallacies of the doctrine of mental discipline and from the field of mental hygiene anent the disintegrating of personality through successive experiences of failure. It has led gradually but now fairly completely to the abandonment of

[2] Bell, E. T.: *Men of Mathematics*. New York, 1937.

requirements in the exact and exacting studies and in place of these a greatly increased emphasis upon the so-called social studies in the belief that these not only are inherently more attractive but will much better equip the learner for the duties of citizenship.

In the interest of retaining as many pupils as possible in our high schools, it may readily be conceded that a relaxation of the more rigorous standards has been a necessity. I have protested for many years against the growing tendency to interpret that necessity as a virtue and to adopt the policy now prevailing of discouraging even competent minds from meeting challenging tasks. For more than thirty years, I have protested against what with increasing clearness has been a quite unjustified generalization of the experiments popularly supposed completely to have invalidated the theory of mental discipline. There is evidence which, in my judgment, cannot be controverted not only that mental discipline may be achieved but that to deprive a learner of the more difficult learning experience is unjust both to him and to society the welfare and progress of which must depend upon disciplined minds. And the assumption that such fields of study as mathematics, physics, and the classical languages offer unique advantages from this point of view can be rationally defended, especially in opposition to the appealing plea for the social studies.

Nor is it impossible to overemphasize the evils of initial failure. The real difficulty arises when failure is permitted to stigmatize the learner as hopelessly incompetent. This attitude has often drawn a specious sustenance from the psychologists' fatalistic and deterministic conception of mentality which was given so wide a vogue by the development of the intelligence tests, but which is now itself well along on the road to partial if not complete discreditment.

Be all this as it may, it is well to remember that a great many men who later achieved a very high measure of distinction were initial failures. The two greatest scientists of the nineteenth century did not promise well in school. One of these —Pasteur—could not finish his first year at the École Normale, but had to go back for further preparation, returning later to graduate, and eventually, after further struggles, to launch upon one of the most brilliant and socially useful of human careers. And it is said that the man who so far heads the list of 20th century scientists was a quite sad failure in the work of the secondary school that he attended, and curiously enough his failure seemed to be most serious in elementary mathematics.

These examples could be multiplied but this is unnecessary here. What I wish to emphasize is the tragedy of stigmatizing early failure as we so often do, and the equally tragic policy of discouraging young people from undertaking tasks that are inherently difficult.

On the other hand, there is no reason for making learning unnecessarily difficult and unnecessarily dull. One of my reasons for favoring the type of training that I have proposed for prospective teachers is that such a training may make them sensitive to the difficulties that young learners —or beginning learners whatever their age—are likely to meet, how these difficulties may be overcome in part at least, and how effort toward overcoming them may be inspired through a recognition of the significance of the materials to be learned, an understanding of their contribution to human progress, and an appreciation of their cost in terms of human effort and initiative and persistence.

What I hope for particularly in connection with mathematics is that such a training will make it possible for a substantial proportion of each generation to share in those precious and important elements in our cultural heritage which mathematics represents and the mastery of which at least in some measure will enable many learners to climb to the high planes of insight and understanding that only some of the most gifted of their predecessors were competent initially to attain.

The Nineteenth Annual Meeting of The National Council of Teachers of Mathematics

ATLANTIC CITY, NEW JERSEY

By EDWIN W. SCHREIBER, *Secretary*

THE 19th Annual Meeting of the National Council of Teachers of Mathematics was held in Atlantic City, New Jersey, February 25–26, 1938. This was the largest of the four Atlantic City meetings—343 registering and some 400 in attendance.

Two simultaneous sessions opened the program, one on "Teacher Training" and the other on "Reports on the Status of Mathematics in the U. S." In the "Teacher Training" section presided over by Dr. Vera Sanford the following papers were presented:

1. "What Professional Training is Needed for Teachers of Arithmetic in the Elementary Schools" by Clifford B. Upton.

2. "The Teacher's Professional Study of Subject Matter in Mathematics" by Wm. C. Bagley.

3. "Training in Mathematics Desirable for Teachers in the Elementary Schools" by Herman M. Cooper.

4. "What Academic Training in Mathematics should be Required of all students in the Three- and Four-year Normal School" by E. H. Taylor, and L. H. Whitcraft.

In the "Reports on the Status of Mathematics in the U. S." section presided over by Mrs. Florence Brooks Miller, the following papers were presented:

1. "Activities of the National Council of Teachers of Mathematics in Tennessee" by F. L. Wren.

2. "Glimpses of Plans and Procedures from Secondary Mathematics Teaching in California" by Edith L. Mosman.

3. "The Status of Mathematics as a Requirement for University Entrance in Nebraska" by A. R. Congdon.

4. "A Report from Connecticut" by Dorothy S. Wheeler.

5. "Certain Curriculum Changes to Meet the New Requirements of the College Entrance Board" by H. G. Funkhouser.

6. "Organization and work of Committees of Men's Mathematics Club of Chicago and Vicinity" by J. T. Johnson.

7. "A Brief History of the National Council of Teachers of Mathematics" by Edwin W. Schreiber (Illustrated with 2″×2″ lantern slides).

At 4:00 P. M. there was a meeting of State Representatives and delegates to the National Council of Teachers of Mathematics.

Evening meeting, 8:00 P. M. Address of Welcome, Dr. Charles H. Elliott, Commissioner of Education of New Jersey. Report of the Work of the Joint Commission on the Place of Mathematics in the Secondary Schools. U. G. Mitchell, Chairman Pro tem, A. A. Bennett, M. L. Hartung, Ruth Lane, C. A. Hutchinson.

The Saturday morning program began with two simultaneous sections, one on "Arithmetic" and the other on "High School Mathematics." The Arithmetic program presided over by H. C. Christofferson was as follows:

1. "Deferred Arithmetic" by Leo J. Brueckner and G. T. Buswell.

2. "Work of the Arithmetic Committee, Progress to Date" by R. L. Morton.

3. Discussion with J. T. Johnson, Leader.

The high school section presented the following program with Miss Martha Hildebrandt presiding:

1. "The Nature and Place of Objectives in Teaching Geometry" by E. R. Breslich.

2. "Geometries" by Mayme I. Logsdon.

3. "Applying Geometric Methods of Thinking to Life Situations" by Elizabeth L. Hall.

4. "An Excursion into the Realms of the Imaginaries" by Luise Lange.

The Discussion Luncheon Saturday noon was a distinct success. 305 attended this session seated at some 32 tables with a discussion leader at each table.

The first section in the Saturday afternoon program concerned Junior High School mathematics with Virgil S. Mallory presiding.

1. "Glimpses from the Baltimore Classrooms" Nanette L. Roche.

2. "A Unit in Mathematical Geography Introducing Rectangular and Polar Coordinates at the Junior High School Level" by E. H. Hildebrandt.

3. "The Unit of Statistics in Ninth Grade Mathematics—Goals, Methods, Difficulties" by Helen Walker.

The second section concerned Senior High School Program with Martha Hildebrandt presiding.

1. "The Evaluation of Teachers of Secondary Mathematics"by Robert H. Morrison.

2. "A Means of Improving the Relation between Secondary-school and College Mathematics" by Ralph Beatley.

3. "The New Attainment Tests of the College Entrance Examination Board and a More Significant Preparation in Mathematics" by J. R. Kline.

At the annual business meeting of the National Council, Secretary Schreiber announced the results of the annual election. President, Dr. H. C. Christofferson, Miami University, Oxford, Ohio; Second Vice-President, Miss Ruth Lane, University High School, Iowa City, Iowa; Members of the Board of Directors, Miss Kate Bell, Lewis and Clark High School, Spokane, Washington; M. L. Hartung, Ohio State University, Columbus, Ohio; Rolland R. Smith, Classical High School, Springfield, Mass.

The annual banquet was attended by 163 and proved to be a very fitting close to the two-day session. The address of the evening was made by Prof. William R. Ransom, Tufts College, on "Take-and-Put Algebra." President Hildebrandt introduced the newly elected officers and directors of the Council, past presidents, and other honored guests. Secretary Schreiber gave his customary analysis of the registration by states, 25 states being represented at this meeting. Professor Reeve made a brief address concerning the National Council and at the close presented our retiring President, Miss Martha Hildebrandt with a small token of esteem from the Board of Directors for her faithful work as President during the last two years. The concluding address was made by our new President, H. C. Christofferson, Miami University, Oxford Ohio.

A splendid mathematical exhibit, presented by the New Jersey teachers, was was very much appreciated.

Meetings of the Board of Directors

Informal Meeting of the Board of Directors, February 24, 1938, 8:00 P. M. Club Room, Hotel Traymore

Present: Martha Hildebrandt, Edwin W. Schreiber, William D. Reeve, Vera Sanford, R. R. Smith, Virgil S. Mallory, B. W. Lidell, Chairman of the Local Committee.

Because of the small number in attendance at this meeting the group discussed informally some of the problems confronting the National Council but no definite action was taken.

Meeting of the Board of Directors, February 25, 1938, 10:30 A. M. and the Adjourned Meeting at 5:00 P. M., Club Room, Hotel Traymore

Present: Martha Hildebrandt, J. T. Johnson, Edwin W. Schreiber, William D. Reeve, Vera Sanford, W. S. Schlauch, M. L. Hartung, R. R. Smith, E. R. Bres-

lich, Leonard D. Haertter, Virgil S. Mallory, H. C. Christofferson, William Betz, Edith Woolsey, Florence Brooks Miller, and Past Presidents C. M. Austin, John P. Everett, Miss Marie Gugle.

Absent: Mary Kelly, Mary A. Potter.

The meeting was called to order by President Martha Hildebrandt and opened with the reading of the minutes by Secretary Schreiber. After discussing several items the minutes were approved as read. The Treasurer's report was presented by Mr. Schreiber and accepted subject to the report of the auditor. Dr. Vera Sanford, auditor, reported that she examined the accounts of Treasurer Schreiber and found them to be correct. (See report published elsewhere.)

It was moved by Mr. Schreiber, seconded by Mr. Christofferson and carried that the President appoint a committee with Mr. Schorling as chairman, the balance of the committee to be chosen with the cooperation of Mr. Schorling, to edit a small pamphlet on "Why Study Mathematics" to be sold for five cents or less per copy.

It was moved by Mr. Reeve, seconded by Mr. Christofferson and carried that we hold a meeting in New York the last of June, 1938. It was moved by Mr. Christofferson, seconded by Mr. Hartung and carried that we recommend to the Budget Committee an appropriation of $300 be set aside for the New York meeting.

It was moved by Mr. Schreiber, seconded by Mr. Reeve, and carried that Mrs. Florence Brooks Miller, Chairman of State Representatives have the same status as a member of the Board as far as expenses to the annual meeting are concerned. It was moved by Miss Sanford, seconded by Mr. Smith, and carried that a vote of appreciation be given to Mrs. Miller for her splendid work as Chairman of State Representatives. In connection with this motion Miss Sanford made the suggestion that in some cases it would be advisable for Mrs. Miller to contact state committees or certain key people before determining a state representative. Miss Sanford also suggested that in some states a committee in place of a state representative might function more advantageously.

It was moved by Mr. Schreiber, seconded by Mr. Reeve, and carried that the President's report, Mrs. Miller's report on State Representatives, and Miss Kelly's report on affiliated societies be accepted with thanks and placed on file. It was moved by Miss Sanford, seconded by Mr. Schreiber and carried that the President send a message of appreciation of the work of Miss Kelly and regret her inability to be present due to illness.

Mr. C. M. Austin, First President of the National Council and State Representative from Illinois, suggested that the $5.00 allowed him for state representative was inadequate in paying for postage and stationery and he feels that if the Board could give more money for this work it would be appreciated by the state representatives. It was moved by Mr. Hartung, seconded by Mr. Mallory and carried that a small committee of the Board be appointed by the President to study the feasibility of having a full-time or part-time person or some other means, to carry on activities of the National Council such as press service, etc. by which more publicity might be secured than at the present time. Miss Sanford suggested that it might be worthwhile sending a few members of our Board to meetings of the National Council of Teachers of English and other National meetings to secure information concerning the running of their meetings, and the manner in which they handled publicity.

It was moved by Mr. Schreiber, seconded by Mr. Mallory and carried that we revise our present ballot procedure and that in the future we publish in the October or November issue of the *Mathematics Teacher* the Official Notice of the annual election and notify at that time that affiliated groups or members may suggest to the ballot committee any prospective candidates. Early in January the official

ballot listing two names for each office and a blank line for an additional name will be sent by government post card to the members of the National Council. It was suggested that the final slate be submitted to the Board of Directors before it was sent to the members and that candidates for the office of president be approached before their names appeared upon the ballot.

It was moved by Miss Sanford, seconded by Mr. Smith, and carried that the Board officially recognize the Arithmetic Committee as appointed by Professor R. L. Morton, Chairman.

It was moved by Mr. Schlauch, seconded by Mr. Mallory and carried that the President appoint a committee to make a study of the feasibility of the National Council contributing something in the way of visual aids such as moving picture films, still films, and slides.

It was moved by Mr. Reeve, seconded by Mr. Betz, and carried that the Secretary write a letter of sympathy to the family of Miss Orpha Worden of Detroit whose recent death was a shock to all of us. It was moved by Miss Sanford, seconded by Mr. Johnson and carried that a letter of sympathy be sent to the family of Miss Mabel Winspear whose death last summer was a lamentable shock to the office of the *Mathematics Teacher*.

Mr. Schreiber presented the matter of publishing a new register of members and after considerable discussion it was moved by Mr. Schlauch, seconded by Mr. Schreiber and carried that we recommend to the Budget Committee that they include in the budget appropriate funds for publishing such a register.

It was moved by Mr. Mallory, seconded by Mr. Christofferson and carried that our next annual meeting be held with the American Association of School Administrators or if necessary, in an adjoining city. It was moved by Mr. Christofferson, seconded by Miss Woolsey and carried that we hold a winter meeting with the A. A. A. S. at Richmond, Virginia and that we recommend to the Budget Committee to set aside $300 for this meeting.

It was moved by Mr. Schlauch, seconded by Mr. Reeve, and carried that the Secretary prepare a 12–16 page pamphlet concerning the National Council to be used for answering inquiries about our organization and that a recommendation to the Budget Committee be made to set aside funds for this pamphlet

It was moved by Mr. Schreiber, seconded by Mr. Hartung and carried that the bibliography on mathematics as suggested by Professor F. L. Wren, be published in a paper edition and edited by the Committee on Official Journal. Mr. Reeve reported that the 1938 yearbook would contain the report of the Joint Commission but that this report might be delayed until late summer or early fall.

It was moved by Mr. Schreiber, seconded by Mr. Johnson and carried that Mr. Reeve, Mr. Schlauch, and Mr. Betz represent the Board in an official capacity concerning the World's Fair to be held in New York City in 1939.

Mrs. Florence Brooks Miller made a brief report to the Board concerning her work as Chairman of the State Representatives. The meeting adjourned.

Meeting of the Board of Directors, February 26, 1938, 4:30 P. M., Stratosphere Room, Hotel Traymore

Present: Martha Hildebrandt, J. T. Johnson, Edwin W. Schreiber, W. D. Reeve, Vera Sanford, W. S. Schlauch, M. L. Hartung, R. R. Smith, Virgil S. Mallory, William Betz, H. C. Christofferson, Edith Woolsey, Ruth Lane.

Absent: Mary Kelly, Mary A. Potter, E. R. Breslich, L. D. Haertter, Kate Bell.

Meeting was called to order by President Hildebrandt who introduced to the Board our new President, Dr. H. C. Christofferson, our new Second Vice-President, Miss Ruth Lane, and three new members of the Board of Directors, Miss Kate Bell, who because of distance

was not able to be with us, Dr. M. L. Hartung, and Mr. Rolland R. Smith.

It was moved by Miss Woolsey, seconded and carried that Miss Martha Hildebrandt be appointed to fill the unexpired term of Mr. Christofferson on the Board of Directors which is for two years, 1938–40.

It was moved by Mr. Schreiber, seconded by Mr. Smith, and carried that the Finance Committee for the year 1938–39 be composed of H. C. Christofferson, Chairman, Martha Hildebrandt, W. S. Schlauch, and the the Budget Committee be composed of W. S. Schlauch Chairman, H. C. Christofferson, and Martha Hildebrandt.

After considerable discussion on the report of the Joint Commission by Mr. Reeve, Mr. Betz, Mr. Hartung, and Mr. Schlauch the meeting adjourned.

Annual Business Meeting of THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, *February 26, 1938, 3:45 P. M., Stratosphere Room, Hotel Traymore*

The meeting was called to order with President Martha Hildebrandt in the chair. She asked for the minutes by Secretary Schreiber on the last annual meeting. It was moved by Mr. C. M. Austin, seconded and carried that the minutes of the 18th Annual Meeting be approved as printed in the May issue of the *Mathematics Teacher*. Mr. Schreiber then gave the Treasurer's report which was approved as read.

Secretary Schreiber then gave the report of the Ballot Committee which was as follows: For President: H. C. Christofferson 565 votes; A. R. Congdon 284 votes. For Second Vice-President: Miss Ruth Lane 511 votes, Frank L. Wren 323 votes. For Members of the Board of Directors: Miss Kate Bell 459 votes; M. L. Hartung 476 votes; U. G. Mitchell 425 votes, Frank M. Morgan 319 votes, Miss Beulah Shoesmith 331 votes; Rolland R. Smith 461 votes. Secretary Schreiber then declared the following duly elected: President, H. C. Christofferson, Miami University, Oxford, Ohio; Second Vice-President, Miss Ruth Lane, University High School, Iowa City, Iowa; Members of the Board of Directors, Miss Kate Bell, Lewis and Clark High School, Spokane, Washington; M. L. Hartung, Progressive Education Committee, Ohio State University, Columbus, Ohio; Rolland R. Smith, Classical High School, Springfield, Massachusetts. Mr. Schreiber reported a total of 868 votes cast, which is the largest number of votes cast at any election in the history of the National Council.

Mr. Reeve reported to the group the significant actions taken by the Board of Directors at its two meetings on the previous day.

Mr. Mallory suggested that we spend more time, money, and energy in the matter of publicity for the National Council. Mr. Reeve reported the death of Professor Tyler and suggests that official notice of same be made by the National Council through the *Mathematics Teacher*.

It was moved by Mr. Wren, seconded by Mr. Congdon and carried that the Board of Directors take under advisement the appointment of a committee to consider the advisability of compiling resolutions concerning the policy of the National Council with reference to the teaching of mathematics. Several other items were discussed but no definite action was taken. The meeting adjourned.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

REPORT OF THE TREASURER

For the Year, February 9, 1937–February 9, 1938

Balance on hand at beginning of year:			
Union National Bank of Macomb, Illinois	$1582.53		
Savings Bank Deposit	2249.51		
New York Telephone Bond, 4½%, Nov. 1, 1939	991.46		
New York Telephone Bond, 4½%, Nov. 1, 1939	547.29		
Commonwealth Edison Bond, 5%, July 1, 1953	938.45		
	6309.24		
Less Net Loss on Book Value of above bonds	7.77		$6301.47
Receipts for the year:			
Mathematics Teacher, W. D. Reeve	$1234.08		
Bureau of Publications, Yearbooks	1974.56		
Bureau of Publications, Kasir No. 385	16.20		
Interest on Bonds	142.50		
Interest on Savings	23.12		3390.46
			$9691.93
Expenditures for the year:			
Chicago Meeting:			
Directors' Expenses	$ 459.69		
Program Speakers	223.25		
Local Committee	74.78	$767.72	
Detroit Meeting		274.66	
Indianapolis Meeting		245.63	
President's Office		25.51	
Chairman State Representatives		260.00	
Chairman Affiliated Associations		5.40	
Arithmetic Committee		258.77	
Secretary-Treasurer's Office:			
Postage and Supplies	$ 271.38		
Stationery	33.84		
Recording Fees and Tax on Checks	1.90		
Bond Expense	10.27		
Secretary Service	300.00	617.39	2455.08
			$7236.85
Balance Sheet, February 9, 1938			
Assets:			(for 1937)
Commercial Bank Deposit	$1270.23		$1582.53
Savings Bank Deposit	2380.53		2249.51
New York Telephone Bond	995.58		991.46
New York Telphone Bond	527.36		547.29
Commonwealth Edison Bond	946.49		938.45
Commonwealth Edison Bond, Bought July 1, 1937 at $1120	1116.66		
	$7236.85		$6309.24
Liabilities:			
None			
Net Worth	$7236.85		$6309.24

(Signed) EDWIN W. SCHREIBER, *Treasurer*

The above report has been audited, and found correct

(Signed) VERA SANFORD, *Auditor*

ATTENDANCE FOR THE 19TH ANNUAL MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, ATLANTIC CITY, NEW JERSEY, FEBRUARY 25–26, 1938

CALIFORNIA:
Berkeley
*Mossman, Edith

COLORADO:
Boulder
*Hutchinson, Charles

CONNECTICUT:
Hartford
*Shanok, Caspar
*Holbrooke, Lillian
*Neal, Beatrice
*Wheeler, Dorothy
New Britain
*Fuller, K. G.
*Weeber, Margaret

DISTRICT OF COLUMBIA:
Washington
*Amig, Margaret
*Dawes, Elizabeth
*Ewin, Mary
*Grubbs, Ethel H.
*Haworth, Ellis
*Judge, Florence
*Phillips, M. R.
*Schult, Veryl
*Stenson, William

ILLINOIS:
Charleston
*Taylor, E. H.
*Heller, H. F.
Chicago
*Breslich, E. R.
*Johnson, J. T.
*Logsdon, Mrs. Mayme
Evanston
*Donalds, Elliot
Leach, Edgar
Macomb
*Edwin W. Schreiber
Maywood
*Martha Hildebrandt
Oak Park
*Austin, C. M.
Princeton
*Smyth, Grace
Winnetka
*Smith, Gene

INDIANA:
Muncie
*Whitcraft, L. H.

IOWA:
Cedar Falls
*Kearney, Dora
Iowa City
*Lane, Ruth

KANSAS:
Lawrence
*Mitchell, U. G.

KENTUCKY:
Lexington
*Mitchell, J. S.
Louisville
*Gregory, Cottele

MARYLAND:
Baltimore
Annig, Catherine
Culley, Alfred E.
*De Ran, Jeannette
Fisk, Marion
Gerbode, Elsa J.
Gibson, A.
*Haferkorn, Marie
Haslup, Norma
Hedeman, Ruth
*Heinaerbridge, Margaret
*Herbert, Agnes
Hough, Katherine
Illich, Mary
Lowe, Margaret
McHale, Helen V.
*Norris, Grover Wm.
O'Donnell, Eleanor
Reese, Ursula
*Roche, Nanette
Roscher, Elsa
*Scarborough, Mary
Sellors, Evelyn
Seymour, Rosalie
Shafer, Esther
*Tames, Katherine
*Taylor, Dorothy
Weiss, Eva
Whelan, Loretta
Reisterstown
Meeks, Anna
Tipton, C. Louise
Ruxton
*Foster, Bess W.
Towson
Mutijka, Elizabeth
Morrison, Helen

MASSACHUSETTS:
Cambridge
*Beatley, Ralph
Boston
*Gifford, Anna
*Stevens, Ernest
Medford
*Ranson, William R.
New Bedford
*Carroll, Mary
Wilbur, Gladys
Springfield
*Smith, Rolland R.
Watertown
Thayer, Helen

MICHIGAN:
Ann Arbor
*Schorling, Raleigh
Detroit
Clifford, Gracey
*Porter, Enos
Flint
*Loss, Nellie
Kalamazoo
*Everett, John P.

MINNESOTA:
Minneapolis
*Woolsey, Edith

MISSOURI:
St. Louis
*Haertter, L. D.
Kirksville
*Jamison, G. H.
*Pemberton, W. S.

NEBRASKA:
Lincoln
*Congdon, Allan R.

NEW HAMPSHIRE:
Exeter
*Funkhouser, H. Gray
Hanover
*Morgan, Frank M.
Plymouth
*Smith, Geneva

NEW JERSEY:
Asbury Park
*Parkhill, Martha
Schmitt, Dorothy
Atlantic City
Brennan, Gertrude M.
Bateman, Josephine
Brown, Fan
Rhodes, Mabel
*Freed, Hubert
*Lidell, Burton W.
Spencer, Mary
*Stanburg, Bertha
Paul, Mabel
Sailor, Anna
Gauntt, Eloise
Washington, Mary
Bayonne
*Burritt, Edna
Bergenfield
*Johnston, Margaret
Bloomfield
Haupin, James
Bridgeton
*Bradway, Minnie
*Carll, B. Eleanor
*Sheppard, Anna
Camden
*Lukens, Marion
*McCobb, Robert
Crawford
*Estabrook, Gladys
Elizabeth
Austin, Bennie
Clark, Elsie
Gorgeses, Mrs. F.
Featherston, Theresa
*Loughren, Amanda
Reichel, Esther
*Wade, Adah
Englewood
*Wildrick, Charles
Fort Lee
*Quinn, Mary
Frenchtown
*Carpenter, Margaret
Glen Ridge
*Ward, Benjamin
Hasbrouck Heights
*Seils, Gertrude

Highland Park
*Hamilton, William
Hillside
McWalters, Lillian
Irvington
Tremper, Clara
Jersey City
*Grossnicke, Foster
Helm, Miss
*Hemenway, Alice
Jensen, Margaret
Kukelski, Mrs. C.
*McMartin, Janet
*Peters, Rachel
Pinkham, Florence
Soman, Frances
Sterling, Alice
Lakewood
Stout, J. Provost
Lawrenceville
*Durell, Fletcher
*Mikesh, G. S.
Leonia
*Smith, Doris
Long Branch
*Tobey, William
Manasquas
Oakley, Chauncey
Montclair
Bolger, Helen
*Clifford, Paul
*Conine, Elmer
Davis, David
Davis, Vera
*Fehr, Howard
*Grauss, Lorraine
*Helserrath, Joseph
*Mallory, Virgil S.
*Stabler, E. Russell
Weinberg, Alan
Upper Montclair
Crystal, Sheva
De Meester, Ruth
*Hildebrandt, E. H. C.
Saller, Daisy
Moorestown
*Bechtel, Blair
*Carr, Wilbur
Walton, Jean
Morris Plains
Lynch, James
Morristown
Barrera, Josephine
*Daly, Dorothy
*Johnson, Abbie
Mountain Lakes
*Schofield, Hayden
Mount Holly
*Sinton, Elizabeth
Newark
*Conkling, Roscoe
*Ayers, N. Howard
Frost, Gladys
Humphreys, Mildred
*MacDonald, Alan
*Owen, Samuel
Quinn, Genevieve
Sanders, Paul
Snavely, Harry
New Brunswick
Boldt, Yetto
McMahon, Susan
*Morris, Richard
*Rickenbrode, F.
*Smith, Mary
*Waring, Agnes
Nutley
Assnuis, Edward
East Orange
Malloy, Lauris
*Norganese
Palmer, Harold
South Orange
Rogers, Frank
Passaic
Arnold, A. S.
*Stiles, Mary
Paterson
Helsenrath, Joseph
*Jaugherty, D. Jr.
*Bedford, Fred
*Bedford, Mrs. Fred
Offhouse, Charles
Paulsboro
Stearle, Winifred
Perth Amboy
Jacobson, Rae
*Kertes, Ferdinand
Plainfield
*Bogart, Charles
Garthwaite, George
Pompton Lakes
Albigese, Leonora
Rahway
*Person, Joseph
Ridgewood
Leach, William
*Powell, Nell
Rockaway
Clamuroo, Phil
Roselle
*Messner, Madeline
Rumson
Barnes, Esther
*Eidel, Ellen
Rutherford
Hitchcock, Edna
Succa Sunna
Morehead, Allan
Summit
*Emerson, Josephine
*Shuttlesworth, Jos.
Tenafly
Ehrentraut, Julia
Toms River
Hoffman, Charles
Trenton
*Colliton, J. W.
Hunsinger, Sanford
Jordan, Max
*Payton, Anne
Vaux Hall
*Zofay, Agnes
Verona
Cook, M. Imogene
Westfield
*Hewitt, Annie
*LePori, Arthur
Melanson, Melvin
*Rogers, Mary

NEW YORK:

Alfred
*Seidlin, Joseph
Bronxville
Greene, Mary
Brooklyn
*Bennett, Pearl
*Lazar, Nathan
*Wallack, Israel
Buffalo
*Crofts, Mary
*Phillippi, Joseph
*Podmele, Theresa
Cortland
*Sueltz, Ben
Garden City
*Griswold, Alice
Geneva
Boughton, Helen
*Cooney, Margaret
Glen Cove
*Richardson, Ruth
Great Neck
Duncan, Ruth
*Maknen, Christine
Ilion
Welden, Flora
Lindenhurst
*Anderson, Alfred
Long Island City
*Byrne, Margaret
Lynbrook
Slauson, Jathryn
Swarthout, Marjorie
Northport
*Lloyd, S. Manford
New Rochele
*Carroll, L. Grace
*Postlewaite, Edna
New York
*Anson, Esther
*Atherton, Charles
*Bergstresser, Clynton
*Brown, Fannie
*Carpenter, Glenda
*Colvin, Zella
*Elliott, Archie
*Greenberg, Ettz
*Hlavaty, Julius
*Kussmann, Ruth
*Loder, L. G.
*McCormack, Joseph
*Mullins, William
*Murphy, Ethel
*Nellenbogen, Marion
*Orleans, Joseph
*Palmer, Marietta
*Reeve, W. D.
*Roll, Rose
*Schlauch, W. S.
*Schutzman, William
*Stoll, Louise
*Sutherland, Ethel
*Torbert, James
*Williams, Robert
Oneonta
*Sanford, Vera
Oneida
*Burma, Frances
Peekskill
*Curtis, Catherine
Riverhead
*Carlin, Grace
Rochester
*Betz, William
*Hall, Mrs. John
*Plean, Brainerd
*Westfall, Bertha
Rockville Centre
Bowe, Mary
*Reeve, Alice
Sayville
*Burgie, Harriett
Smith, Violet

E. Setauket
*Fog, Harold
Staten Island
*Wiseman, John
Syracuse
*Carroll, I. S.
*Dence, Clarence
Valley Stream
*Henry, Ettz
Woodmere
*Babcock, Lester

OHIO:
Athens
*Benz, H. E.
*Emmert, R. W.
*Morton, R. L.
Cincinnati
*Sister Mary Henrietta
* Sister Mary Edward R.S.M.
Cleveland
*Miller, A. Brown
Columbus
*Fawcett, Harold
*Gugle, Marie
*Hartung, M. L.
*Cahoon, G. P.
Oxford
*Christofferson, H. C.
Shaker Heights
*Miller, Florence Brooks

OKLAHOMA:
Fahnquak
*Peeler, Harry

PENNSYLVANIA:
Abington
*Cosmay, Beatrice
Bryn Mawr
*Woodruff, J. S.
California
*Salisbury, E. G.
Elkins Park
*White, Robt.
George School
*Evans, Walter
Indiana
*Malachek, Joy
Kingston
*Adams, James
*Bruce, Edgar
Millersville
*Gress, Elizabeth
Newton Square
*Mast, J. Earl
Philadelphia
Abrahamson, Norman
*Bechtel, John
*Carl, Elmer
*Constable, Mary
*Darnell, Alice
Gottlieb, Herbert
*Groff, Margaret
*Hallowell, J. B.
*Holbrook, Alice M.
Isard, Walter
Kratz, Paul
*Reilly, Margaret
*Rorer, Dr. J. T.
*Rosengarten, Elizabeth
Rubin, Daniel
*Shallenberger, Helen
*Stokes, C. N.
*Taylor, Joseph
*Wood, Elizabeth
East Stroudsburg
*May, Jonas
Trucksville
*Saxe, Esther
Wayne
Lutz, Rose
West Chester
*Holland, Mary

RHODE ISLAND:
Newport
*Coffin, Almira
Lowe, Dr. H. T.
Providence
*Adkins, J. B.
*Talley, Wm. W.

TENNESSEE:
Nashville
*Wren, Frank L.

VIRGINIA:
Hampton
*Atkinson, Robt.

WEST VIRGINIA:
Morgantown
*Wheat, H. G.

* Signifies member
* 235 or 68.5%
108 or 31.5%

ATTENDANCE BY STATES

California	1	Maryland	33	Ohio	12
Colorado	1	Massachusetts	8	Oklahoma	1
Connecticut	6	Michigan	5	Pennsylvania	33
Dist. of Columbia	9	Minnesota	1	Rhode Island	4
Illinois	12	Missouri	3	Tennessee	1
Indiana	1	Nebraska	1	Virginia	1
Iowa	2	New Hampshire	3	West Virginia	1
Kansas	1	New Jersey	135		
Kentucky	2	New York	66	Total	343

VOTES BY STATES

Alabama	4	Maine	1	Oklahoma	29
Arizona	2	Maryland	8	Oregon	8
Arkansas	3	Massachusetts	28	Pennsylvania	43
California	17	Michigan	45	Rhode Island	4
Colorado	12	Minnesota	20	South Carolina	4
Connecticut	8	Mississippi	5	South Dakota	5
Delaware	0	Missouri	16	Tennessee	32
Dist. of Columbia	8	Montana	4	Texas	14
Florida	8	Nebraska	42	Utah	2
Georgia	4	Nevada	0	Vermont	4
Idaho	2	New Hampshire	4	Virginia	9
Illinois	96	New Jersey	36	Washington	7
Indiana	25	New Mexico	1	West Virginia	2
Iowa	31	New York	105	Wisconsin	15
Kansas	74	North Carolina	10	Wyoming	6
Kentucky	6	North Dakota	2		
Louisiana	0	Ohio	56	Total	868

Deferred Arithmetic*

By Leo J. Breuckner
University of Minnesota

In this discussion I wish to differentiate between two aspects of the content of the arithmetic curriculum. One aspect may be designated as *social arithmetic*, which includes those learning experiences whose primary purpose it is to help the pupils to understand the functions and applications of numbers in the affairs of their daily lives and in activities of the community. In these activities the emphasis is on adding meaning and significance to social as well as quantitative elements of experience rather than on teaching pupils how to manipulate number processes. The other aspect of arithmetic may be designated as *computational arithmetic*. In this work the primary purpose is to present the number facts and processes in a systematic way for the purpose of developing skill in computation and number manipulation. In the ideal program these two aspects of arithmetic, the social and the computational, should supplement each other. However, it is commonly recognized today that in the past the tendency has clearly been to over-emphasize the formal computational aspect of the subject and to place relatively little stress on the social-informational functions of the subject.

It has been repeatedly revealed by surveys of the work of the schools that large numbers of pupils have been required to repeat the work of a grade because of failure to master what were regarded as the essentials of arithmetic, usually expressed in terms of computational arithmetic. Many factors contributed to this result. Emphasis placed on formal drill in number processes and failure to develop an understanding of number meanings meant that for many pupils the work was a routine, often meaningless, experience. A faulty psychology based on the idea that young children can learn very readily through drill and repetition led the schools to place too heavy a load on the lower grades. Processes now known to be too difficult for children in the various grades to master, were presented too early in the curriculum. Often ineffective teaching procedures were employed which failed to recognize individual differences in the rates of learning. Materials of instruction were also much less efficiently organized than at present.

As a consequence of these surveys, numerous measures were taken to reduce the number of non-promotions at the various grade levels. In some systems parallel track plans were devised which would enable groups of pupils to progress at different rates. In other systems plans of ability grouping were tried out. In others attempts were made to differentiate the content of the curriculum and to adapt it to the needs of the pupils. In still others, for example, in Winnetka, a curriculum was devised which was organized in such a way that pupils could progress at practically individual rates in the skill phases of learning. None of these plans solved the problem.

Increasingly it is being recognized that the traditional concept of a "grade," defined as a body of skills to be mastered, as a basis of school organization, must be completely changed along lines which recognize the developmental process as the soundest basis of organization. Recent investigations, for example in the state of New York, show that the present tendency is to reduce to a minimum non-promotions which require a pupil to repeat the work of a grade. In some places non-promotion has been practically eliminated in favor of a policy of what we may call "uninterrupted continuity." Such a pro-

* Paper read at the annual meeting of the National Council of Teachers of Mathematics at the Hotel Tragmore in Atlantic City, N. J., on Saturday, February 26, 1938.

gram requires that instruction in arithmetic be so organized that the gait of the work is adapted to the stage of the child's progress. Since the arithmetical ability of children grows at different rates it is evident that a plan of instruction must be utilized, especially that dealing with computational arithmetic, which is in large part individualized. The elements of such a plan have long been used in cities like Detroit, Winnetka, and Minneapolis, where practice exercises are so organized that pupils in one class can work on differentiated assignments according to their needs.

The various tendencies briefly outlined above have had important effects on curriculum organization in arithmetic. In 1928 Washburne demonstrated the fact that there were wide variations in the grade placement of most of the major topics in the curriculum. For example, some schools presented long division in grade three, others did not introduce it until grades five or six. At the present time the situation is even more chaotic. Wide variations are found in the grade placement of topics in the most recent textbooks. One series introduces one phase of division of fractions in grade four, others delay it until late in grade six. These and similar trends show that attempts are being made to adjust the curriculum more nearly to the stages in the child's development, but because of lack of agreement as to what should be done, a situation best described by the term chaotic exists.

Several groups of important investigations on the teaching of arithmetic have contributed findings that have led schools to make changes in the organization of the curriculum. One group of studies dealt with the effect of postponing or deferring the teaching of arithmetic in the primary grades. Included in this group are the studies by Ballard in 1912, Taylor in 1916, Wilson in 1930, and Benezet in 1935–36. In these studies formal arithmetic instruction was withheld in one group and administered as usual in another group. At the end of the experimental period, the comparative achievements of the two groups were measured. In each case the experimenter recommended the postponement of "formal" arithmetic—Ballard for two years or the age of seven, Taylor for one year, Wilson for two years, and Benezet until grade 5.

On the basis of these and other studies the plan of eliminating formal arithmetic instruction from grades one and two, sometimes also grade three, has been adopted by a considerable number of school systems. In some systems there is not even an approved plan of informal or incidental arithmetic. Such a procedure fails to recognize certain very important facts about the studies referred to above. A careful reading of the reports of these four experiments shows that while formal practice on computational processes was postponed in the experimental groups, there was a great deal of use made in these classes of various kinds of activities, games, projects, and social situations through which the child was brought into contact with numbers and given the opportunity to use them informally in meaningful ways. It is especially clear in the studies by Wilson and Benezet that arithmetic was not in fact postponed at all. It is evident that what happened in these two studies was that computational arithmetic was replaced by what I called earlier in this paper, social arithmetic. In each study the plan was to emphasize number meanings, to develop an understanding of the ways in which number functions in the daily lives of children both in and out of school, and to develop what is called number "readiness" for the more formal work to follow. From these studies the conclusion should be drawn not that arithmetic should be postponed to grade three or four, but that the introduction of social arithmetic in the first few grades does not result in any loss in efficiency when the formal computational aspect of the work is introduced later on, say in grade three.

A group of studies that has an important bearing on the question of when to

begin instruction in arithmetic deals with the number knowledge children possess at the time they enter the school. Such well known studies as those of Buckingham and McLatchy, Woody, Beito, and others, have quite conclusively demonstrated that on the average children at that age have a much larger knowledge of number and its applications than was believed likely ten years ago. This knowledge must be acquired through informal contacts with number in the activities of daily life. However in all studies very wide variations were found in the knowledge possessed by the different pupils. Some possessed practically no information on the subject, while others had a fairly advanced background. From these studies the conclusion may be drawn that it is an unsound policy to postpone all arithmetic instruction for one or two years, since if this is done many pupils will be quite unready for the formal work to be introduced at that point. To make certain that the basic and essential meanings are established the school should plan a series of well-chosen units of work in the field of social arithmetic which will make it certain that the pupil has the necessary experiences to insure readiness for the formal work on computations. It is not safe to leave this development to the incidental contacts with numbers in the experience of the school since it is quite probable in many cases that the curriculum may not provide frequent enough contact with numbers to be effective. This has been revealed by the study by Hanna reported in the *Tenth Yearbook of the National Council of Teachers of Mathematics*.

There are some who will not accept the above recommendation on the desirability of emphasizing social arithmetic rather than formal computational arithmetic in the primary grades. They wish to begin at once with formal computational arithmetic in these grades. They base their opinion on a group of studies which show that children at this grade level can in fact be taught number processes with good success. For example, Knight and Behrens, Beito, and others, found that pupils in grades one and two can be taught the addition and subtraction combinations. MacGregor showed that in Scotland where pupils begin arithmetic at the age of five years, the achievements of eleven year old pupils are considerably higher than children of the same age in this country. It is difficult to account for the superiority of the Scottish children. It may be that instructional procedures and materials are more efficient than those used in this country. In any case, it is evident that a much higher degree of mastery of skills is possible than is achieved in many places in this country. An important factor is that the program of arithmetic instruction which produced these results begins at the age of five. The Committee of Seven investigation reported by Washburne in 1928 showed that sixth grade pupils who started arithmetic in grade one made higher scores on a battery of tests in the various processes than were made by sixth grade pupils who began arithmetic in grades two or three. Similarly sixth grade pupils who began arithmetic in grade two made higher scores than the pupils who began the work in grade three. However, the differences in computational ability found were small. No evidence of differences between the groups in tests of social aspects of arithmetic is available.

The questions may be raised, Were the small differences resulting from one or two years more of work in formal arithmetic worth the price? Could the time have been spent to greater advantage on social arithmetic or other important aspects of learning? Any answer to these questions must be wholly speculative, since the evidence on which to base any conclusion is lacking. My own view on this matter is that we should systematically evaluate all of the general outcomes resulting from various kinds of programs in primary arithmetic, in which are emphasized in varying ways and degrees the contributions of social and computational

arithmetic. It is quite evident that in the past we have evaluated programs in primary arithmetic almost wholly from the standpoint of the computational arithmetic learned by the pupils. The possibilities along the other line should be carefully investigated in the light of the broader conception of the functions of arithmetic instruction, now generally accepted by the schools.

To this point we have devoted our attention largely to the work in the primary grades. Let us now turn our attention to the work at other levels of the school. It is obviously essential to consider the work here also from the standpoint of both social and computational arithmetic. From the point of view of social arithmetic, the problem of the organization of the curriculum is to a considerable extent similar to that for the social studies. The present tendency in that field is to consider the program as a continuous one beginning at the time the child enters school and continuing until the end of the period of formal education and even beyond. The curriculum is to be so selected that it will contribute at all points to the development of a well-rounded intelligent individual, disposed and able to participate in an effective way in the efforts made by society to improve the existing social order. It is held that the ideals, attitudes, insights, abilities, and other kinds of outcomes of this work are not the product of any one grade, but are the result of a cumulative process to which the work at all levels contributes. The units of work at each level are to be selected in such a way that they are adapted to the interests, needs, and abilities of the pupils at that level; and because it has been demonstrated that they contribute to the achievement of the ultimate objective of the school. The work of no grade would be repeated by a pupil because of failure to achieve certain standards such as are now used to measure progress or achievement.

It seems to me that a similar program could be developed in the field of social arithmetic. I can visualize a sequence of rich, vital social arithmetic units that will contribute to the general objectives that are set up for all education, and in particular the broad social functions of arithmetic. The emphasis in these units of work would be on developing an understanding and an appreciation of the significance of the elements of the social process now commonly dealt with in arithmetic, such as money, banking, measuring, taxation, insurance, buying and selling, elements of simple economics, wages, geometric design, and similar items. These topics would have to be taught in units of work carefully adapted to the interests, needs, and abilities of the pupils. There would be a general sequence of these units which at succeeding levels would contribute to a richer, broader treatment of the topics and the consideration of their more difficult aspects. No pupil need be required to repeat any of this work as would be done under the conventional policy of non-promotion. To provide for differences in the mental capacity of pupils, the work in the various units would need to be so organized as to provide for tasks of a wide variety of levels of difficulty adapted to the variations among the pupils.

The basic elements of such a program in social arithmetic are now found in many places. Minneapolis, Cleveland, the Speyer School, Bronxville, and other school systems are undertaking the systematic study of possible social arithmetic units and evaluating their effectiveness. Several such units are now in print. Various publishing companies are publishing excellent informational readers, bulletins, and other instructional materials essential of the development of an enriched program in social arithmetic. Textbooks also reflect this trend.

It is unfortunately true that at the present time we lack any comprehensive experimentally established basis for determining what to teach along this line at the various levels of the school. Preliminary steps have been taken along a

few lines by the Committee of Seven to determine the optimum points at which to teach any unit of social arithmetic, but this evidence is meager and does not touch many of the social aspects of the work. This field offers a wealth of opportunity for experimental study, which might be sponsored by this Society.

The desirability of integrating as far as is possible the work in social and computational arithmetic suggests a possible basis for the gradation of units in social arithmetic at levels above the primary grades, namely the selection of units of work for a particular level of the school that will give ample opportunity to apply the number processes which can most effectively be presented at that level. For example, it is possible to select units of work rich in applications of fractions which can be used as the basis of the work in the grade or grades in which fractions are taught. It would obviously be an unwise practice deliberately to select units of work involving complicated fractions for the primary grades because the pupils could not handle them at this stage of development. Such units would be much more suited to the fifth or sixth grade level. This plan of selecting units applying the processes assigned to the grade has been followed by Harap in his three studies of the learning of number processes in activity units. He selected the units of work because they provided ample opportunity to use the processes assigned to the grades in which he was experimenting.

If one accepts this approach as one basis of selecting units in social arithmetic for the various grades, the questions now arise, what are the optimum points at which to present the various number processes? What basis shall be used in the organization and arrangement of the contents of the work? I think that it can be successfully maintained that in the past the organization and sequence of the materials of the curriculum has largely been determined by the logic of the number system. We started in with the work with processes in whole numbers, then took up fractions, then decimals and percentage, and ended with the more general phases of mathematics. This basis of arrangement overlooked the important factor of the relative learning difficulty of the various processes. It did not establish a valid basis for determining the point at which a given process can most readily and effectively be learned by the pupils. The result was the more or less blind groping by schools seeking to solve problems about gradation which led to the present confusion. Grade placement was determined by prevailing practice and not by experimentally established facts.

Undoubtedly the most important series of investigations that has yet been conducted on the problem of grade-placement of computational processes in arithmetic is the group of studies conducted by the Committee of Seven, under the leadership of Dr. Washburne. It will not be possible within the time limits set to discuss these studies fully. Suffice it to say that they indicate that the traditional logical arrangement of processes is not the same as an arrangement based on their learning difficulty. For example, addition of like fractions is clearly easier than long division with two-place divisors. Yet we usually teach the latter before the former. Addition and subtraction of decimals are easier to master than the same processes in common fractions, yet decimals are taught after common fractions. It also seems evident that many processes, for example long division, and division of fractions have, in the past, been introduced into the curriculum considerably before the average pupil could learn them successfully and retain the skills. The Committee set up minimum and optimum mental ages at which to present each process. Scores on basic foundations tests were also made available. This information is of real value, insofar as it is dependable, in determining the pupil's readiness for a topic and the point at which to teach it. The need of foundation training is clearly recognized by the Committee of Seven.

If one is willing to accept these findings, a considerable reorganization of the arithmetic curriculum is inevitable. The effects of the work of this committee are revealed by the changes that are being made in courses of study throughout the country, and in the newer textbooks in arithmetic. It is important that we bear in mind the fact that the findings of the Committee of Seven substantiated practices that had been adopted in various centers even before the investigations of the Committee were begun. These schools can now proceed with more assurance since their plan is in accordance with some of the major findings of this committee.

The findings of the Committee of Seven should not be adopted as final, because of some possible limitations of these studies. The ratings for difficulty of processes are undoubtedly true for the conditions under which the investigations were conducted. The findings might differ considerably in schools where the materials are more effectively constructed, the teaching is more competent and better supervised, and the basic training of the pupils is more adequate. For example, Grossniekle has recently shown that, under competent supervision with carefully organized materials, fourth grade pupils having a mental level of 10 years can be taught long division with good success. This level is more than two years below the level set by the Committee of Seven. The reliability of the findings of the Committee has also been questioned by some, for example Brownell, on the ground that the tests, both of mental ability and achievement, were not adequate measures of these functions. The criterion of learning set up by the Committee has also been challenged by several investigators. The point is made that many pupils can learn a given process at levels considerably below those set up by the Committee. Furthermore, it would be desirable to secure information about the learning difficulty of various elements in each major process, so that the teaching of the whole process could be more effectively distributed over a number of grades. Such a plan would undoubtedly be superior to the presenting of the whole process at the time when, according to the Committee, even its most difficult elements can be mastered by the pupils. The recognition of these possible limitations indicates that there is a need of further experimentation dealing with the problem of gradation, an enterprise which the National Council of Teachers of Mathematics should endorse. The Council should assist interested groups to secure the funds needed to carry on the necessary research.

The picture of the arithmetic curriculum I have been trying to present can be briefly summarized as follows: The core of the arithmetic curriculum should be social arithmetic. This core should consist of a series of carefully selected units of work of demonstrated value adapted to the interests, needs and stage of the development of the pupils, and intended to give them a rich social insight and understanding of the functions of numbers in daily life, and to enable them to participate more effectively in the affairs of a changing industrial democratic society. The units of work should vary from locality to locality, although the basic principles underlying their selection should be the same in all communities. Through these units of work the pupil should also be led to see the functional value of the various number processes and be given ample opportunity to apply them in a meaningful way. These processes should be assigned to grade levels at which they can most readily and effectively be learned. This will require some re-arrangement of the subject matter. Any drill work to insure mastery of the number processes should be organized in such a way as to provide adequately for differences in the stage of development of the pupils and their rates of growth.

◆ THE ART OF TEACHING ◆

Whither Mathematics?

By WINSTON E. ROMIG, *Windber, Pa.*

THERE are encouraging signs that the teachers of mathematics are beginning to realize that far-reaching reforms are necessary in their domain if mathematics is not to be forced from the high school curriculum. There are indications that there is a growing consciousness that no amount of patch-work revamping of methods and subject-matter; no amount of piecemeal philosophizing will turn the trick. There seems to be, at last, some disposition on the part of the teachers themselves to apply caustics strong enough to reach to the roots of the ills which afflict us. One such indication is the article by Mr. Joseph Seidlin, "What Shall We Do With Our Unfit" in THE MATHEMATICS TEACHER for February, 1938, for that article was strong medicine. This paper is in part a reply to Mr. Seidlin, and in part an appeal for some constructive thinking and comment from the persons who are most affected—the teachers of mathematics.

Mr. Seidlin's criticisms on the present teaching of mathematics were severe but on the whole I think that they were surely justified. However, when he suggests that the proper point at which to begin the needed reform is a drastic elimination of the "unfit" from our ranks, I am afraid that he is attempting to put out the fire by pulling down the house. His method might, indeed, put an end to the conflagration, but would it save the property? It seems to me that it is of doubtful expediency to decimate the teachers, at least until it is ascertained beyond doubt that the teachers are personally most to blame for the present state of affairs. It is here that we reach the main point at issue. I think Mr. Seidlin is wrong when he implies that the most fundamental cause of the admittedly poor showing of mathematics in our schools is lack of intelligence and lack of preparation on the part of the teachers, although I am ready to admit that we are sadly deficient in both. I am of the opinion that a program of reform, predicated for the main part upon the improvement of the teaching staff, would get us nowhere until some of the muddled thinking which chokes what intelligence and efficiency we have is cleared up.

The teacher of mathematics finds himself at present pretty much in the position of Mr. Cobb's famous horseman who mounted his nag and rode off in all directions. He is ordered or invited to go in half a dozen different directions at once, with the result that he has lost the points of the compass and knows not which way to turn. After fifteen years of trying to teach high school mathematics I still find myself in a woeful state of confusion as to basic aims and objectives. I feel that our present futility in the teaching of mathematics arises fundamentally from a lack of orientation. Let me try to summarize, as briefly as possible, some of the forces which are responsible for this confusion:

1. There is a school of thought which looks upon mathematics as being essentially a vocational subject. The persons who subscribe to this school expect thats any one who has finished his twelve grades should be able to step into the store or office and, without further training, be able to perform, quickly and accurately, the computations which are needed in the carrying on of business. The fabulous man in the street probably belongs to this school. If he could dictate

the aims and objective of the teaching of mathematics he would emphasize the mastery of processes in "practical" applications, with special emphasis on speed and accuracy.

2. There are those who still hold, more or less consciously, to the doctrine of formal discipline. To them, mathematics is a subject preeminently qualified to train the mind in logical thinking, and, some would add, the only subject now in the schools which is so qualified. The proponents of this school of thought believe that mathematics may maintain its position by paying attention to "identical elements" of thought situations, "teaching for transfer" and so forth. They still hope that it is possible to "teach pupils to think."

3. There has recently been much discussion about the "coordination" and "integration" of mathematics with the other subjects. The coordinators have tacitly abandoned the position that mathematics is of great intrinsic value. They take the attitude that mathematics must justify its retention in the schools by its usefulness in obtaining results and clarifying thought in other fields.

4. Closely associated with the point of view of the integrationists is still another which is somewhat more radical. It is the idea that the specialized teaching of mathematics should be abandoned altogether, and mathematical processes taught incidentally in connection with the other subjects. Those who hold to this point of view live in the hope that out of a load of individual bricks the pupil will be able to build a unified structure.

5. Diametrically opposed to the two points of view cited directly above is that of the reactionaires who are enamoured of "pure mathematics." The followers of this school believe that the function of the teaching of mathematics in the elementary and secondary schools is to lay a foundation upon which may be erected the beautiful temple of the "queen of the sciences." The feelings of this group are essentially aesthetic. They have the artist's horror of those who hold more utilitarian views although they have been forced to yield considerable ground before the latter.

6. Finally, there is over all of this the dead hand of tradition which compels the teacher to prepare his pupils for college, although he knows that only a small per cent of them will go to college, and a small per cent of that small per cent will engage in studies which require a further pursuit of mathematics when they get there.

These are some of the currents of thought which I feel exerting their forces upon me as a teacher, some with more, some with less effect. The teacher of mathematics is assailed on all sides with a clamor to move in this particular direction, or to turn his back and go in a direction directly opposite. His superintendent assumes one point of view, his principal another, and his supervisor a third. The expert who is invited in to survey and make recommendations advocates a course of action which is directly contradicted by the prefaces to the text books which he uses. Finally he is driven to scramble together some sort of philosophy for himself which may bear no resemblance whatever to that of the teacher in the next room. I submit that even the best prepared and most intelligent teachers would have but little chance to make any progress toward really good teaching if they are never to stand together upon some common ground of action. I reiterate my belief that the most fundamental cause of the present poor showing of mathematics in our schools is lack of orientation.

What I have to say next I frankly admit is based upon a guess—a guess which I hope and believe is properly rooted in observation and experience. The few meditations which follow are thrown out with the hope that they may stimulate some thought and discussion by teachers of mathematics, and that out of this dis-

cussion may come some basis upon which the National Council of Teachers of Mathematics may do some research toward establishing a common ground upon which the teachers may stand together.

My guess is that mathematics will ultimately be driven to justify its existence in the public schools upon the same grounds as all other subjects; that is, upon its usefulness to the entire student body, which is to say, in effect, its usefulness to the entire population. I know that this utilitarian view runs counter to the prejudices of many of our leaders in the field of mathematics, but I must run the risk of having Plato and Euclid hurled at my head. After all we are dealing with twentieth century Americans, not with the Greeks of two thousand years ago. At any rate, it may be of interest to posit this assumption and see where it leads. These, I believe, are the main implications:

1. Mathematics in the future will have to be taught for its social values, for these are the only values which are certain to reach all the people. But what are the social values of mathematics? I think they are, in the main, protective. I surmize that we shall have to teach the pupil how to protect himself against the frauds and fallacies that are now foisted upon him through his ignorance of the uses of mathematics. Perhaps the mathematics curriculum of the future will deal with such problems as: a critical comparison of the sizes of containers; the necessity for a mathematically accurate grading of goods, with plainly marked labels; a discussion as to whether an increase in the number of cylinders or tubes necessarily makes a better car or radio, worth the increased cost; the detection of the fallacy in a tax program which attempts to balance the budget by soaking the rich; a calculation of the costs, financial and social, of heavy installment buying. Call this socialized mathematics, consumer mathematics, or what you will, it seems to me that it is the only kind of mathematics which is of vital importance to every man, woman, and child in the country. It seems to me, also, that there is enough depth and latitude in such a program to satisfy even the most ardent mathematician.

2. A second implication lying in my assumption is the possibility of breaking through the conventional boundaries, of whatever kind, which now so badly cramp the teaching of mathematics. By following a socialized program we should be able at once to burst through the water-tight compartments of mathematical thought, and to disregard the divergent aims and claims which at present so much distract us. Pursuit of the solution of a social problem in numbers would necessarily cut across many sections of formal mathematics, and hard-and-fast lines between subjects would have to be disregarded. And why not? There are no valid reasons why many of the concepts of algebra, geometry, and the calculus should not be taught at a much earlier age than they are now. Likewise with the fields of application. It is tradition only which confines school-boy applications of mathematics to the fields of commerce and engineering. Under a socialized program the teacher would feel free to draw upon that compartment which would best yield a fitting method for the problem in hand, and to step into that field of application which was of most interest at the moment. No doubt the business man will grumble if we neglect to stress speed and accuracy in mechanical processes, but he has grumbled anyway. The purists will be discomfited, but perhaps the children will be interested. Teachers in allied fields will probably not be able to look to us for much specific help, but I think they will not be much the worse off. The gain in freedom to teach should prove incalculable. In any event, if my major premise is correct, we will have but little choice. We will have to select and teach those aspects of mathematics which the people value, or we will have to move out of the schools altogether.

3. I see one more implication arising from the foregoing; an implication which may seem radical or subversive. I do not insist upon it, I merely suggest it as a logical conclusion to my main premise. I think the greater portion of mathematics teachers will probably have to regard themselves as engaged in teaching social rather than natural science. This leads to a number of further implications—too many to be analyzed at this time; but I would mention just two of them: (a) Teachers of mathematics will need a quite different preparation than teachers in service have had, or prospective teachers are now getting. (b) Specialists will be needed to care for the interests of vocational and college preparatory groups and to cooperate with the teachers of the other natural sciences. As for the main body of mathematics teachers, their job will be to teach the mathematics of general appeal to the great mass of unspecialized pupils who have little need for and no interest in the other kind.

I have tried in this short space to peer a little way into the fog which surrounds the teaching of mathematics. If I have progressed but a little way toward a common ground upon which the teachers of mathematics can stand together, I have succeeded in my intent. If not, I may hope that

"—the dry bones littered by the way
May still point giants toward their golden prey."

PLAYS

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Mathesis. Brownell, Ella. XX, Dec. 1927.
The Eternal Triangle. Raftery, Gerald, XXVI, Feb. 1933.

The Mathematics Club Meets. Pitcher, Wilimina Everett, XXIV, April 1931.

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IN OTHER PERIODICALS

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

1. Barnett, I. A., "Geometrical Constructions Arising From Simple Algebraic Identities." *School Science and Mathematics*, 38: 521–527. May, 1938.

The purpose of the paper is to give some supplementary work in geometrical constructions which may be utilized at the end of a course in plane geometry. Methods are described for using the straight edge and compasses in the construction of $a+b$, $a-b$, ab, $a \div b$, and in the solution of the quadratic equation, $x^2-px+q=0$.

Four examples are also given "to illustrate how simple algebraic identities lead to interesting properties of configurations consisting of straight lines and arcs of circles." The following identities are illustrated: (1) $\sqrt{a^2}=a$; (2) $\sqrt{a}\cdot\sqrt{a}=a$; (3) $(ab)\div c=(a\div c)b$; (4) $\sqrt{ab}=\sqrt{(ab)}\cdot 1$.

2. Campbell, Alan D., "Some Weaknesses in Mathematical Training." *National Mathematics Magazine*, 12: 347–350. April, 1938.

After enumerating many weaknesses the writer found in the mathematical background and knowledge of graduate students, he proceeds to enumerate those changes he would like to see made in the textbooks and in the mathematical curricula of secondary schools and colleges.

3. Curtis, H. B., "A Graphical Solution of the Cubic." *National Mathematics Magazine*, 12: 325–326. April, 1938.

A description of a method of solving a cubic equation graphically in which the cubic itself need not be graphed, for all solutions are made to depend on the graph of the cubical parabola, $y=x^3$.

4. Dobbs, W. J., "Morley's Triangle." *The Mathematical Gazette*, 22: 50–57. February, 1938.

It is well known that the trisectors of the angles of a triangle intersect in points which form the vertices of an equilateral triangle, generally called "Morley's Triangle." The writer points out, however, that since each of the angles of the original triangle may be trisected each in three ways, there are really 27 "Morley" triangles to be considered. He then proceeds to show that 18 of these 27 triangles are equilateral.

5. Fletcher, W. C., "Euclid." *The Mathematical Gazette*, 22: 58–65. February, 1938.

Many interesting comments on questions relating to the axiom of parallels and the sequence of theorems.

6. Lennes, N. J., "The Foundation of Arithmetic." *The American Mathematical Monthly*, 45: 70–75. February, 1938.

A critical analysis of the foundations of arithmetic in terms of the theory of sets.

7. Mullen, Frances A., "The Math Star." *Chicago Schools Journal*, 19: 169–172. March-April, 1938.

An excellent description of the interest and enthusiasm that can be aroused by a school mathematics magazine. Although the article deals with a magazine published at the Fenget High School, Chicago, Illinois, it can be highly recommended as a general treatment of the topic "The Value of School Mathematics Magazine." The method of organizing such a magazine, the kind of articles and problems that will interest the youngsters, and other relevant topics are treated with great thoroughness.

The writer seems to imply that there are very few such publications in the United States. She may be interested to know that there are at least a dozen mathematical magazines published by the students of mathematics in the high schools of New York City. Other large cities will most likely yield similar information.

8. "New Geometry for Germany." *The Mathematical Gazette*, 22: 73–76. February, 1938.

Three views are expressed on the desirability of following the decree of the German minister of the interior to subdivide the quadrant into 100 parts. The participants are, F. B. Pidduck, E. Borel (in French), and D. A. Sadler.

9. Nordgaard, Martin A., "Sidelights on The Cardan-Tartaglia Controversy." *National Mathematics Magazine*, 12: 327–346. April, 1938.

A fascinating historical chapter on an old topic that is ever new.

10. Nygaard, P. H., "Co-ordination of Elementary Arithmetic Teaching with the Methods of High School Mathematics."

School Science and Mathematics, 38: 370–375. April 1938.

The author points out "how the instructional methods used in grade school arithmetic might be modified so that pupils can make the transition to high school mathematics with less difficulty." The comments deal with the following topics:

1. The arrangement used by pupils in writing their solution of verbal, or thought, problems;
2. The methods used in adding and subtracting fractions;
3. The terminology used in division problems;
4. The advisability of introducing algebra into the curriculum of the upper elementary grades.

The writer's analysis is clear and the recommendations are, in the main, practical. Unfortunately he deals only with the desirable modifications to be made in the teaching of *arithmetic*. He fails to point out the importance of the converse problem—that of changing certain *algebraic* procedures and algorisms in order to conform with those generally, almost universally, accepted in arithmetic.

Apropos the last point, see *The Mathematics Teacher*, 27: 53–59, January, 1934, and the bibliography mentioned there.

11. Olds, Edwin G., "A Fresh Start." *National Mathematics Magazine*, 12: 290–293. March, 1938.

An interesting description of the methods used to interest a class of "repeaters" in Differential Calculus.

12. Sanders, S. T., "Mathematics, Servant to Humanity." *National Mathematics Magazine*, 12: 320–322. April, 1938.

"The centre of the vast cosmos of general human interests is under its steadily growing influence. Most pressing problems of human need are being solved by its help, and the more closely drawn together are the individuals of human society by apparent ultra-rational forces, the more pervasive and seem to become the ministration of mathematics to the needs of man."

13. Spear, Joseph, "Mathematics—To Reason Not Just to Do." *School Science and Mathematics*, 38: 402–410. April, 1938.

In order to discourage mechanical operations with fractions, the method of unity is used exclusively. " . . . In dealing with fractions the only operation permitted, in order not to change the value of a fraction, is to multiply it by unity or to divide it by unity. The unity may be disguised in various forms in order to accomplish the desired end, but multiplication or division by the factor of unity is the only privilege permitted." The advantages of the application of the method to various operations with fractions in arithmetic and algebra are clearly indicated.

In solving equations the use of the words "cancellation" and "transposition" is discouraged. "In all that has been said here, the fundamental idea is to teach the student to understand the reasons for doing things."

14. Taylor, E. H., "Mathematics for a Four-Year Course for Teachers in the Elementary School." *School Science and Mathematics*, 38: 499–503. May, 1938.

It is pointed out that (1) "students entering teachers' colleges are deficient in both the mechanics and the understanding of arithmetic; (2) the offerings and requirements in mathematics for elementary school teachers are on the decline; (3) the success of mathematics throughout the school system depends upon the quality of work done in the elementary school." It is urged that the National Council of Teachers of Mathematics "as well as the Mathematical Association of America and the American Mathematical Society have an important work in attempting to improve the preparation of teachers of mathematics in the elementary field."

15. Tripp, Myron, O., "The Teaching of Financial Mathematics." *School Science and Mathematics*, 38: 542–549. May, 1938.

The object of the author is "to show how the usual problems of finance may be solved by the use of a few elementary principles, thus avoiding the long list of formulae ordinarily given in the average treatise on the subject." Problems are worked out in great detail which deal with the following topics: amount at simple interest, amount at compound interest, the time required for a certain amount to double itself at compound interest, the rate earned by investing in U. S. Savings Bonds, annuities, amortization, depreciation, perpetuities and capitalization.

"Certainly the prospective teacher of mathematics must develop some perspective in dealing with topics taken up by any class, and hence we must conclude that the mathematics of finance is a very essential vitalizing element for the high school teacher."

16. Yates, Robert C., "A Trisector." *National Mathematics Magazine*, 12: 323–324. April, 1938.

In 1875 M. Laisant proposed a contrivance for trisecting an angle. That mechanism contained a sliding device. In this article the author shows that, by the addition of a single bar and the rearrangement of some others, we may discard the undesirable slide and have a pure linkage containing only joints. Diagrams are included both of the original and of the modified trisectors.